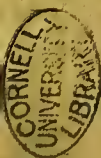


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UNITED STATES COAST SURVEY.

CARLILE P. PATTERSON, SUPERINTENDENT.

METHODS, DISCUSSIONS, AND RESULTS.

METEOROLOGICAL RESEARCHES

FOR THE USE OF

THE COAST PILOT.

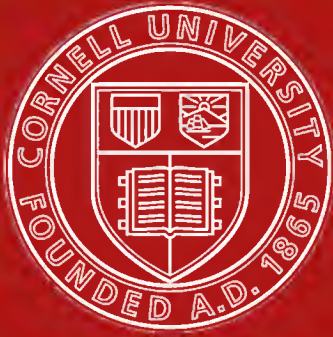
PART I.



WASHINGTON:

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UNITED STATES COAST SURVEY OFFICE,
Washington, D. C., July 21, 1877.

The great storms which yearly traverse with terrible energy some portion of the vast extent of sea-coast that bounds the United States on the Atlantic and Pacific Oceans and the Gulf of Mexico have caused in the aggregate the loss of many thousands of lives and the destruction of an immense amount of property.

The most violent of these storms are the cyclones that originate commonly in the Atlantic Ocean near the equator. They pass over the West Indies, curve around by the Gulf of Mexico and Florida, and then sweep the entire coast of the Atlantic in a northeasterly direction.

The frequency of recurrence, and the marked force exerted along the same general course, suggest that these cyclones result from the operations of laws that control the general motions of the atmosphere, and that some understanding in regard to the cause and time of their occurrence might be gained by research, extended so as to include an area commensurate with the developed phenomena; but, until now, it is believed that no attempt directed to that end has been made. Any knowledge in advance respecting the direction and rate of motion of these storms would be of incalculable benefit to commerce and navigation, as well as to the people living along the sea-coasts. Their investigation has, therefore, been undertaken in the hope that the exact knowledge of the configuration of the coast and of its dangers, given to the mariner in the Coast Pilot, may in time be supplemented by information concerning the atmospheric disturbances to which the same region is subject. Mr. William Ferrel, to whom this discussion has been intrusted, and whose previous studies and special ability peculiarly qualify him for the proposed research, presents, in the following paper, the results of an investigation of the mechanics and the general motions of the atmosphere. This preliminary inquiry will be followed by his investigation of the effects of various disturbances that are local, as compared with those upon which general motions depend. Such disturbances give rise to local cyclones, due to unequal distributions of temperature in the northern and southern hemispheres. It will be shown also that local disturbances of equilibrium are the occasion of progressive cyclones.

The principles and results developed in the leading part of the discussion will, in a separate paper, be applied for perfecting the formulæ and methods to be used in the determination of barometric heights; and, as all the general principles applicable to atmospheric motions probably apply also to those of the ocean, the discussion will be supplemented by a chapter on the subject of ocean currents.

These researches will embody conclusions from the most important parts of a memoir published by Mr. Ferrel in 1859 and 1860 in the "Mathematical Monthly." The memoir appeared necessarily as small detached papers, separated by other matter, in two volumes of that journal, and hence the views of the author attracted little notice until extracts from his memoir were quoted in the publications of the United States Signal Service. The reader of the first paper by Mr. Ferrel upon this subject will notice a few slight changes; but it was to be expected that, in a matter so complex, additional years of study would offer both new and improved views.

CARLILE P. PATTERSON,
Superintendent.



METEOROLOGICAL RESEARCHES.

BY WILLIAM FERREL.

PART I.

ON THE MECHANICS AND THE GENERAL MOTIONS OF THE ATMOSPHERE.

CHAPTER I.

GENERAL EQUATIONS OF THE MOTIONS AND THE PRESSURE OF THE ATMOSPHERE.

1. In making out equations of condition for the motions and pressure of the atmosphere or the sea, where the part under consideration comprises the whole or a considerable part of the earth's surface, it is very important that the rotation of the earth on its axis should be taken into account. This can be most conveniently done by referring each particle at any time to three rectangular co-ordinates whose directions are fixed in space, and thus making out equations between the motions, forces, and pressures for each of these directions. These co-ordinates thus become a function of the earth's rotation and the motion of the particle relatively to the earth's surface; and by transforming the rectangular to polar co-ordinates, and expressing the equations in functions of the latter, we obtain equations containing very important terms, depending upon the earth's rotation on its axis.

2. Let x, y , and z be three rectangular co-ordinates having their origin at the center of the earth, x corresponding with the axis of rotation; also let—

V = the potential of the attractive force of the earth;
 P = the pressure of the fluid; and
 k = its density:

then $D_x V$, $D_y V$, and $D_z V$ are the accelerating forces arising from the earth's attraction, and $\frac{1}{k} D_x P$, $\frac{1}{k} D_y P$, and $\frac{1}{k} D_z P$ those arising from the pressure of the fluid in the reverse directions respectively of x, y , and z ; and hence we have, for the equations of the absolute motions of the fluid, regarding the center of the earth at rest,—

$$(1) \quad \left\{ \begin{array}{l} D_t^2 x + D_x V + \frac{1}{k} D_x P = 0 \\ D_t^2 y + D_y V + \frac{1}{k} D_y P = 0 \\ D_t^2 z + D_z V + \frac{1}{k} D_z P = 0 \end{array} \right.$$

Putting $P = 0$, these become the equations of a projectile.

3. Let—

r = the distance from the earth's center;
 θ = the polar distance;
 φ = the longitude;
 n = the angular velocity of the earth's rotation on its axis.

We then have—

$$(2) \quad \left\{ \begin{array}{l} x = r \cos \theta \\ y = r \sin \theta \cos (nt + \varphi) = r \sin \theta \cos \omega \\ z = r \sin \theta \sin (nt + \varphi) = r \sin \theta \sin \omega \end{array} \right.$$

by putting, for brevity, $nt + \varphi = \omega$, and making the origin of t such as to make $\sin (nt + \varphi)$ vanish in the plane of x, y .

From these expressions of x, y , and z , we get—

$$\begin{aligned} D_t x &= \cos \theta D_t r - r \sin \theta D_t \theta \\ D_t y &= \sin \theta \cos \omega D_t r + r \cos \theta \cos \omega D_t \theta - r \sin \theta \sin \omega D_t \omega \\ D_t z &= \sin \theta \sin \omega D_t r + r \cos \theta \sin \omega D_t \theta + r \sin \theta \cos \omega D_t \omega \end{aligned}$$

Taking the second derivatives, we get—

$$(3) \begin{cases} D_i^2 x = \cos \theta D_i^2 r - 2 \sin \theta D_i r D_i \theta - r \cos \theta (D_i \theta)^2 - r \sin \theta D_i^2 \theta \\ D_i^2 y = \sin \theta \cos \omega D_i^2 r + 2 \cos \theta \cos \omega D_i r D_i \theta - 2 \sin \theta \sin \omega D_i r D_i \omega + r \cos \theta \cos \omega D_i^2 \theta \\ \quad - r \sin \theta \cos \omega (D_i \theta)^2 - 2 r \cos \theta \sin \omega D_i \theta D_i \omega - r \sin \theta \sin \omega D_i^2 \varphi - r \sin \theta \cos \omega (D_i \omega)^2 \\ D_i^2 z = \sin \theta \sin \omega D_i^2 r + 2 \cos \theta \sin \omega D_i r D_i \theta + 2 \sin \theta \cos \omega D_i r D_i \omega + r \cos \theta \sin \omega D_i^2 \theta \\ \quad - r \sin \theta \sin \omega (D_i \theta)^2 + 2 r \cos \theta \cos \omega D_i \theta D_i \omega + r \sin \theta \cos \omega D_i^2 \varphi - r \sin \theta \sin \omega (D_i \omega)^2 \end{cases}$$

Since x , y , and z are functions of r , θ , and φ , we have—

$$(4) \quad \begin{cases} D_x V = D_r V \cdot D_x r + D_\theta V \cdot D_x \theta + D_\phi V \cdot D_x \varphi \\ D_y V = D_r V \cdot D_y r + D_\theta V \cdot D_y \theta + D_\phi V \cdot D_y \varphi \\ D_z V = D_r V \cdot D_z r + D_\theta V \cdot D_z \theta + D_\phi V \cdot D_z \varphi \end{cases}$$

We also have—

$$r^2 = x^2 + y^2 + z^2$$

$$\tan \theta = \frac{\sqrt{x^2 + y^2}}{x}$$

$$\tan \omega = \frac{z}{y}$$

From these, we get—

$$D_x r = \frac{x}{r} = \cos \theta$$

$$D_y r = \frac{y}{r} = \sin \theta \cos \omega$$

$$D_z r = \frac{z}{r} = \sin \theta \sin \omega$$

$$D_x \theta = -\frac{\sqrt{y^2 + z^2}}{r^2} = -\frac{\sin \theta}{r}$$

$$D_y \theta = \frac{xy}{r^2 \sqrt{y^2 + z^2}} = \frac{\cos \theta \cos \omega}{r}$$

$$D_z \theta = \frac{xz}{r^2 \sqrt{y^2 + z^2}} = \frac{\cos \theta \sin \omega}{r}$$

$$D_x \varphi = 0$$

$$D_y \varphi = -\frac{z}{y^2 + z^2} = \frac{\sin \omega}{r \sin \theta}$$

$$D_z \varphi = \frac{y}{y^2 + z^2} = \frac{\cos \omega}{r \sin \theta}$$

By means of these equations, equations (4) give—

$$(5) \quad \begin{cases} D_x V = \cos \theta D_r V - \frac{\sin \theta}{r} D_\theta V \\ D_y V = \sin \theta \cos \omega D_r V + \frac{\cos \theta \cos \omega}{r} D_\theta V - \frac{\sin \omega}{r \sin \theta} D_\phi V \\ D_z V = \sin \theta \sin \omega D_r V + \frac{\cos \theta \sin \omega}{r} D_\theta V + \frac{\cos \omega}{r \sin \theta} D_\phi V \end{cases}$$

By putting P for V in (4), we obtain in like manner—

$$(6) \quad \begin{cases} D_x P = \cos \theta D_r P - \frac{\sin \theta}{r} D_\theta P \\ D_y P = \sin \theta \cos \omega D_r P + \frac{\cos \theta \cos \omega}{r} D_\theta P - \frac{\sin \omega}{r \sin \theta} D_\phi P \\ D_z P = \sin \theta \sin \omega D_r P + \frac{\cos \theta \sin \omega}{r} D_\theta P + \frac{\cos \omega}{r \sin \theta} D_\phi P \end{cases}$$

By substituting the values of the first members of equations (3), (5), and (6) in equations (1), and then multiplying both members of the equations respectively by $\cos \theta$, $\sin \theta \cos \omega$, and $\sin \theta \sin \omega$, and adding, we obtain the first of the following equations. Again, multiplying them respectively by $r \sin \theta$, $-r \cos \theta \cos \omega$, and $-r \cos \theta \sin \omega$, and adding, we obtain the second of those equations. Finally, multiplying the last two respectively by $r \sin \theta \sin \omega$ and $-r \sin \theta \cos \omega$, and adding, we get the last of the following equations:—

$$(7) \quad \begin{cases} \frac{1}{k} D_r P = -D_t^2 r + r(D_t \theta)^2 + r \sin^2 \theta (n + D_t \omega) D_t \varphi + r n^2 \sin^2 \theta - D_r V \\ \frac{1}{k} D_\theta P = -r^2 D_t^2 \theta - 2r D_t r D_t \theta + r^2 \sin \theta \cos \theta (n + D_t \omega) D_t \varphi + r^2 n^2 \sin \theta \cos \theta - D_\theta V \\ \frac{1}{k} D_\phi P = -r^2 \sin^2 \theta D_t^2 \varphi - 2r \sin^2 \theta D_t r D_t \omega - 2r^2 \sin \theta \cos \theta D_t \theta D_t \omega - D_\phi V \end{cases}$$

4. Let us now put—

h = the height above the earth's surface ;
 u = linear distance south ;
 v = linear distance east ;
 g = the accelerating force of gravity.

By regarding the cosine of the angle between the directions of r and h , which is extremely small, equal to unity, and neglecting the small terms depending upon the motions of the fluid, which are multiplied into the sine of the very small angle between the directions of r and h , and putting for $D_t \omega$ its equal, $n + D_t \varphi$, we shall have—

$$\begin{aligned} \frac{1}{k} D_h P &= -D_t^2 h + \frac{(D_t u)^2}{r} + \sin \theta (n + D_t \varphi) D_t v + r n^2 \sin^2 \theta - D_h V \\ \frac{1}{k} D_u P &= -D_t^2 u - 2 D_t h D_t u + \cos \theta (2n + D_t \varphi) D_t v + r n^2 \sin \theta \cos \theta - D_u V \\ \frac{1}{k} D_v P &= -D_t^2 v - 2 \sin \theta (n + D_t \varphi) D_t h - 2 \cos \theta (n + D_t \varphi) D_t u - D_v V \end{aligned}$$

5. In the case in which the fluid is at rest relatively to the earth's surface, we have—

$$\begin{aligned} \frac{1}{k} D_h P &= r n^2 \sin^2 \theta - D_h V = -g \\ \frac{1}{k} D_u P &= r n^2 \sin \theta \cos \theta - D_u V = 0 \\ \frac{1}{k} D_v P &= -D_v V = 0 \end{aligned}$$

Hence, in the case of the motions of the fluid, we have—

$$(8) \quad \begin{cases} \frac{1}{k} D_h P = -D_t^2 h + \frac{(D_t u)^2}{r} + \sin \theta (n + D_t \varphi) D_t v - g \\ \frac{1}{k} D_u P = -D_t^2 u - 2 D_t h D_t u + \cos \theta (2n + D_t \varphi) D_t v \\ \frac{1}{k} D_v P = -D_t^2 v - 2 \sin \theta (n + D_t \varphi) D_t h - 2 \cos \theta (n + D_t \varphi) D_t u \end{cases}$$

6. In these equations, there are several terms which are so small that they may be neglected in all cases without sensible error. The value of the term $D_t^2 h$ is of the same order, in comparison with g , as the rate with which an ascending or descending particle of air is accelerated or retarded in comparison with the rate with which a falling body in vacuum is accelerated; since, putting $P = 0$ in the first of the equations (8), we get $g = -D_t^2 h$. In all ordinary conditions of the air, the vertical velocities are so small, and these velocities are accelerated or retarded so gradually, that the value of $D_t^2 h$ corresponding to these accelerations or retardations, in comparison with the rate of acceleration of a falling body, is so small that it may be neglected. In the case of a tornado, the value of this term might be sensible through some part of the height of the atmosphere; but, as the velocity of an ascending body cannot be accelerated in one part of its ascent without being retarded in another part, the integration of this term through the whole height of the atmosphere would be naught, and hence it would not affect P at the surface of the earth.

If any part of the atmosphere has a vertical acceleration or retardation of velocity equal to g in 100 seconds, or a little less than two minutes, then the pressure or value of P arising from this part of the atmosphere is affected the one-hundredth part; and, in such a condition of the atmosphere, in determining heights from barometric pressure, the results would be seriously affected. But such conditions could only take place under some extraordinary disturbances of the atmosphere, when any such determinations would not be attempted. It is seen from the first of (8) that the pressure is diminished when the ascending velocity is accelerated or descending velocity retarded, and *vice versa*.

7. We have, neglecting $D_t \varphi$ in comparison with n ,—

$$\frac{\sin^2 \theta (n + D_t \varphi) D_t v}{g} = \frac{r \sin^2 \theta n D_t \varphi}{g} = \frac{r n^2}{g} \cdot \frac{D_t \varphi}{n} \sin^2 \theta = \frac{1}{289} \sin^2 \theta \frac{D_t \varphi}{n}$$

But $D_t \varphi$, the angular eastward motion of the air relatively to the earth's surface, is always very small in comparison with n , and hence, even at the earth's equator, where $\sin^2 \theta = 1$, the ratio between $\sin^2 \theta (n + D_t \varphi) D_t v$ and g is extremely small, and the former may in all cases be neglected in connection with the latter. The value of the term $\frac{(D_t u)^2}{r}$ is evidently very much smaller than $\sin^2 \theta (n + D_t \varphi) D_t v$, and hence its effect is always entirely insensible.

In the last two of equations (8), the terms containing $D_t h$ as a factor may both be neglected in comparison with the terms which follow them, since the vertical velocity $D_t h$ is always very much smaller than the horizontal velocities $D_t u$ or $D_t v$, where these latter are such as to give a sensible value to the terms containing them as factors.

8. The density of dry air is as the pressure and inversely as the amount of absolute temperature. But the atmosphere always contains a certain amount of aqueous vapor which is lighter than air, and affects the density of the atmosphere in proportion to the amount of this vapor contained in it. This effect is expressed by $(1+f(e))$ in the denominator of the expression for the density, in which e is the relative amount of aqueous vapor contained in the air. We can therefore put—

$$(9) \quad k = \alpha P$$

in which, putting α_0 for the value of α when $t = 0$ and $e = 0$,—

$$(10) \quad \alpha = \frac{273^\circ}{(273^\circ + t)(1 + f(e))} \alpha_0 = \frac{1}{(1 + 0.3663 t)(1 + f(e))} \alpha_0$$

The average value of e varies with the temperature, but is very different at different times and in different localities for the same temperature. As an average value of $f(e)$ for all localities and seasons, we can put—

$$f(e) = 0.00154 + 0.000341 t^*$$

With this value of $f(e)$, we get—

$$(11) \quad \alpha = \frac{\alpha_0}{1.00154 + 0.004 t}$$

* See a paper by Dr. J. Hann, entitled "Zur barometrischen Höhenmessung," in ⁴ Band LXXIV der Sitzb. der kaiserlichen Akademie der Wissenschaften."

This latter expression of α may be used in all cases in which the preceding equations are applied to the atmosphere generally, in which local and temporary deviations must be neglected; but, in local applications, the former must be used, the value of e being determined from observation for the particular time and locality.

9. So far we have neglected to consider the effect of friction, which is an important element entering into equations where the motions of fluids, either elastic or inelastic, are concerned, and one which is most difficult to treat. In the preceding equations, therefore, we must have a term to represent the resistance which each particle suffers from friction, and this term cannot be expressed by any function of the velocity simply, as is sometimes supposed, but it depends rather upon the differences in the velocities of the different strata, and upon the differences of pressure. If a stratum lie between two other strata, all having the same velocity, it suffers no resistance from friction, however great the velocity may be; and the same is the case where the relative velocities of the strata are such that the action of the stratum above upon the intermediate stratum is exactly equal to the reaction of the lower one upon it in the contrary direction. This may require the relative velocities of the different strata to be different on account of the differences in the amount of pressure, and it, no doubt, requires them to increase as the pressure diminishes, that is, with the height. The amount of resistance, therefore, which any particle suffers, requiring extraneous forces to overcome, is generally an unknown quantity, and all that we can do, therefore, is to introduce unknown functions into the equations representing the resistances from friction in the directions of the co-ordinates, and leave these to be determined approximately, where it can be done, from a comparison of the final results deduced from the equations with observation. If we, therefore, put F_u and F_v for the forces acting in the directions respectively of u and v necessary to overcome the resistances of friction, and substitute for k its value in (9), we get from (8), neglecting the insensible terms pointed out in §§ 6 and 7,—

$$(12) \quad \left\{ \begin{array}{l} D_h \log P = -g \alpha \\ D_u \log P = -\alpha D_t^2 u + \alpha \cos \theta (2n + D_t \varphi) D_t v - \alpha F_u \\ D_v \log P = -\alpha D_t^2 v - 2\alpha \cos \theta (n + D_t \varphi) D_t u - \alpha F_v \end{array} \right.$$

In the case of a homogeneous fluid, it is readily seen that we must put P for $\log P$ and k instead of α

The value of g is nearly constant; but, when great accuracy is required, it must be regarded as a function of h and θ , and we may put—

$$(13) \quad g = g' \left(1 - \frac{2h}{r} + 0.00284 \cos 2\theta \right)$$

in which g' is the value of g at the sea-level and on the parallel of 45° .

10. By regarding g as independent of h , equation (12) gives, by integration,—

$$(14) \quad \log P' - \log P = \alpha' \left(gh + f(h) \right) + \alpha' \int_u \left(D_t^2 u - \cos \theta (2n + D_t \varphi) D_t v + F_u \right) \\ + \alpha \int_v \left(D_t v + 2 \cos \theta (n + D_t \varphi) D_t u + F_v \right)$$

in which P' and α' are the values of P and α respectively at the earth's surface, and $f(h)$ is a small function of h depending upon the decrease of temperature with the height, which may always be neglected, especially when h is small, except in accurate hypsometrical determinations. Where common tabular logarithms are used, the last number of this equation must be multiplied into the modulus $M = 0.4342945$.

The preceding expression gives the difference of the pressure between any two assumed points. If these two points are in the same vertical, the terms depending upon $D_t u$ and $D_t v$ vanish, and the expression is confined to the first term; but, if these points are in verticals a considerable distance apart, the integration of the last two terms, depending mostly upon the earth's rotation and the motions of the atmosphere relative to the earth's surface, may give a considerable difference of pressure at the sea-level, or for any two assumed points at equal heights above it.

11. Where P and P' are in the same vertical, we get from (14), by neglecting the small term $f(h)$ in connection with $g h$, and regarding g constant,—

$$\begin{aligned} D_u \log P' - D_u \log P &= g h D_u a' \\ D_v \log P' - D_v \log P &= g h D_v a' \end{aligned}$$

By means of these equations, we get from (12)—

$$(15) \quad \begin{cases} \frac{1}{a'} D_u \log P' = -D_t^2 u + \cos \theta (2n + D_t \varphi) D_t v - F_u + g h D_u \log a' \\ \frac{1}{a'} D_v \log P' = -D_t^2 v - 2 \cos \theta (n + D_t \varphi) D_t u - F_v + g h D_v \log a' \end{cases}$$

At sea-level, we have $h=0$, and consequently the last term of these equations vanishes, and they then give the gradients of barometrical pressure in the directions of u and v , depending upon the motions of the atmosphere at the earth's surface, friction, inertia, and the earth's rotation on its axis. In this case, the small neglected function $f(h)$ also vanishes.

12. If we put h' for the value of h belonging to a stratum of the atmosphere of equal density, or pressure, we shall have, for this stratum, $D_u P = 0$ and $D_v P = 0$; and we get in this case from the first of (14), neglecting the small function $f(h)$ in comparison with $g h$,—

$$\begin{aligned} D_u \log P' &= g h' D_u a' + g a' D_u h' \\ D_v \log P' &= g h' D_v a' + g a' D_v h' \end{aligned}$$

With these equations, we get from (15), by putting $h=0$,—

$$(16) \quad \begin{cases} g D_u h' = -D_t^2 u' + \cos \theta (2n + D_t \varphi) D_t v' - F_u' - g h' D_u' \log a' \\ g D_v h' = -D_t^2 v' - 2 \cos \theta (n + D_t \varphi) D_t u' - F_v' - g h' D_v' \log a' \end{cases}$$

in which u' and v' are the values of u and v at the earth's surface. These equations give the gradients of the strata of equal pressure in the directions of u and v ; and, by integration, they give the differences of level of any two points in such a stratum. This, at the earth's surface, when $h'=0$, depends simply upon the motions of the atmosphere at the earth's surface, friction, and inertia; but the last terms of these equations show that these gradients are increased or diminished, as the case may be, in proportion to the height.

13. From (12) we get, for the part depending upon the earth's rotation,

$$\begin{aligned} D_u \log P &= 2 a n \cos \theta D_t v \\ D_v \log P &= -2 a n \cos \theta D_t u \end{aligned}$$

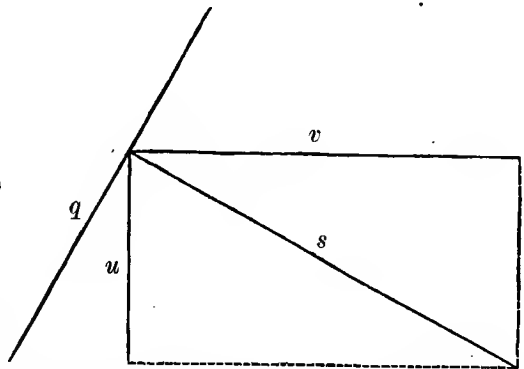
If we now put s for the resultant of u and v , and q a perpendicular to s on the right-hand side of the direction of motion, we shall have—

$$\begin{aligned} D_q \log P &= D_u \log P \cdot D_q u + D_v \log P \cdot D_q v \\ &= D_u \log P \cdot \cos \frac{q}{u} - D_v \log P \cdot \sin \frac{q}{u} \\ D_t s &= D_t u \sin \frac{q}{u} + D_t v \cos \frac{q}{u} \end{aligned}$$

From these and the two preceding equations, we get—

$$(17) \quad D_q \log P = 2 a n \cos \theta D_t s$$

The direction of s is entirely arbitrary, and $D_t s$ represents velocity in that direction, and $D_q \log P$ represents an ascending gradient in the direction of q to the right of the direction of s , and consequently the force depending upon the earth's rotation which causes this gradient, expressed by the last member of (17), is a force acting in that direction, and is positive in the northern hemisphere, and the contrary in the southern,



according to the sign of $\cos \theta$. Hence, in whatever direction a body moves upon the surface of the earth, there is a force arising from the earth's rotation which tends to deflect it to the right in the northern hemisphere, but to the left in the southern hemisphere.

This important principle, useful in explaining so many of the relations between the motions and the barometric gradients of the atmosphere, was first published in my former paper on this subject in the "Mathematical Monthly" in the year 1860. It is a generalization of the principle upon which the theory of the trade winds has been based, according to which this deflecting force, arising from the earth's rotation, takes place only in the case of motions north or south. But, by the true and more general principle above, it is seen that this deflecting force is exactly the same for motions in all other directions.

Influenced by the usual theory upon this subject, observers have imagined that they have seen evidences of a tendency in rivers running north and south to wear away the banks, and also to deposit drift-wood on the right- rather than on the left-hand side, and likewise a tendency in the cars of railroads extending north and south to be thrown off the track on the right- rather than the left-hand side, while in the cases of rivers or of railroads extending in other directions no evidences of such effects could be seen. But we now know that if any sensible effects of this sort arise from this deflecting force in the case of rivers or of railroads running north or south, the very same effects must take place where they run in any other direction.

The amount of this force as deduced above, from the true principles of mechanics, is exactly double of that which has been obtained from the erroneous principle adopted by Hadley, and brought down through text-books to the present time. This latter principle assumes that the moving body in approaching or receding from the earth's axis must retain the same linear motion east or west, whereas, by the principle of the preservation of areas, which must hold in this case, the linear motion east or west must be increased in the former case and diminished in the latter in such proportion as to make the deflecting force double of that given by the principle of equal linear east or west motions for all distances from the earth's axis. This matter has been explained in detail in "Nature," vol. v, p. 384.

14. The accelerating force in the direction of q which is adequate to produce a gradient represented by $D_q P$ is—

$$(18) \frac{1}{k} D_q P = \frac{1}{a} D_t \log P = 2 n \cos \theta D_t s = \frac{2 \cos \theta}{r n} \cdot r n^2 D_t s = \frac{2 \cos \theta}{r n} \cdot \frac{1}{289} g \cdot D_t s = \frac{2 \cos \theta}{289} \cdot \frac{D_t s}{r n} \cdot g$$

The coefficient of g in this expression shows the ratio between this deflecting force and the force of gravity.

Let us put at sea-level, on the parallel of 45° ,—

$r = 6366252^m$	$\log r = 6.8038838$
$n = \frac{2\pi}{(23 \times 60 + 56) 60} = 0.000072924$	$\log n = -5.86287$
$g = 9^m.805307$	$\log g = 0.9914612$
$r n = 464^m.25$	$\log r n = 2.66675$

With $D_t s = 13.889$, which is equal to a velocity in the direction of s of 50^{km} per hour, and with $\theta = 45^\circ$, corresponding to the parallel of 45° , we get $\frac{1}{6839} g$ for the accelerating force in the direction of q arising from the earth's rotation, and hence the lateral pressure of a body moving with the assumed velocity in any direction on that parallel is equal to $\frac{1}{6839}$ of its weight. In the southern hemisphere, where $\cos \theta$ is negative, of course the lateral pressure is in the contrary direction, that is, to the left of the direction of motion. Where a body is free, this deflecting force produces motion in a curved line. In the case of a fluid, as air or water, this force causes a disturbance of the static level surface, and the amount of gradient resulting from it is expressed by the coefficient of g in (18). For instance, with the assumed velocity above and on the parallel of 45° , the gradient would be one meter in the distance of 6839 meters.

If a river has a velocity of 5^{km} per hour, the ascending gradient to the right of direction in the

northern hemisphere is one meter in 6839 meters; and hence, if the river is one kilometer in width, the water stands about $\frac{1}{68}$ of a meter higher on the right than on the left bank.

15. In the case of the atmosphere, in which the gradient is usually measured by the differences of barometric pressure, we get from (9) and (18) for this gradient—

$$(19) \quad D_t P = 0.00014595 a P \cos \theta D_t s$$

In order to have a numerical expression of the gradient in terms of the pressure and temperature, and independent of the density, it is necessary to determine the value of a_0 in (11), from which we thus obtain the value of a corresponding to any given temperature t , and that of a' corresponding to the value of a at the earth's surface, which occurs in the most of the preceding expressions, and of which, therefore, it is important to have a determination.

The value of a_0 is the value of a in (9) belonging to dry air, with the temperature at zero (32° F.). If we put l equal to the height of a homogeneous atmosphere of temperature 0° , and pressure, measured by the height of the barometrical column, of $0^m.76$, we have for such an atmosphere—

$$P = g k_0 l = g k' \times 0^m.76$$

in which k_0 is the density of dry air under the assumed pressure, and k' the density of mercury. Putting $k_0 = a_0 P$ (9), we get, since the heights of the homogeneous atmosphere and mercury are inversely as their densities—

$$1 = g a_0 l = g a_0 \frac{k'}{k_0} \times 0^m.76$$

With $k' = 13.6001$ and $k_0 = \frac{1^*}{772.9}$, we finally get—

$$(20) \quad a_0 = \frac{1}{g l} = \frac{1}{g \times 7989^m}$$

With this value of a_0 , we get from (11) the approximate value of a for any given temperature, to be used in (19) for determining the barometric gradient belonging to the deflecting force arising from the velocity $D_t s$. For the temperature of 0° , a in (19) becomes a_0 ; and, with the preceding value of a_0 , we get in this case—

$$(21) \quad D_t P = 0.000000014162 \cos \theta D_t s$$

in which $D_t s$ must be expressed in meters per second.

If we wish to express the gradients by the change in millimeters belonging to the distance of a mean degree of the meridian, equal 11111111 millimeters, we must multiply the second member above by 11111111, and then, representing the gradient by G , we get—

$$(21') \quad G = 0.15893 \cos \theta D_t s$$

16. In many cases, it is necessary to have the equations of the motions and pressures in terms of polar co-ordinates, in which the pole does not coincide with the pole of the earth's axis.

* Prof. F. A. P. Barnard, *Metric System*, p. 171, has obtained, from the average of the weights of a cubic inch of dry air given by several authorities, 0.0012228315 for the specific gravity of dry air at a temperature of 62° F. and a barometric pressure of 30 inches. This, reduced to the temperature of 0° C. and barometric pressure of $0^m.760$, gives the value above.

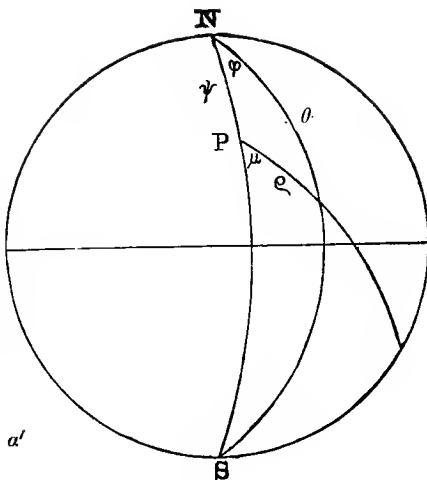
Let—

- ψ = the arc between the pole of the earth's axis and the new pole of the co-ordinates;
- ρ = the distance in arc from the new pole;
- μ = the angle between ρ and the meridian;
- u = linear distance in the direction of ρ ; and
- v = linear distance in a direction perpendicular to ρ .

In the case in which the earth has no rotation on its axis, the pole of the co-ordinates in the preceding equations can be taken at pleasure, and therefore, instead of N, the pole of the earth's axis in the annexed figure, we can put it at P, and hence by putting ρ, μ, u , and v for θ, φ, u , and v respectively, we get from (15) in this case, by putting $n = 0$,—

$$\frac{1}{a'} D_u \log P' = -D_t^2 u + \cos \varphi D_t \mu D_t v - F_u + g h D_u \log a'$$

$$\frac{1}{a'} D_v \log P' = -D_t^2 v - 2 \cos \varphi D_t \mu D_t u - F_v + g h D_v \log a'$$



We must now add to these equations the terms arising from the deflecting forces depending upon the earth's rotation belonging to the velocities $D_t u$ and $D_t v$, which are forces in directions respectively perpendicular to u and v . From (17) we get, for the parts of $\frac{1}{a'} D_u \log P'$ and $\frac{1}{a'} D_v \log P'$,—

$$\frac{1}{a'} D_u \log P' = 2 n \cos \theta D_t v$$

$$\frac{1}{a'} D_v \log P' = 2 n \cos \theta D_t u$$

Adding the right-hand members of these equations to those of the two preceding ones, we get for the equations in the case in which the earth rotates on its axis—

$$(22) \quad \begin{cases} \frac{1}{a'} D_u \log P' = -D_t^2 u + (2 n \cos \theta + \cos \rho D_t \mu) D_t v - F_u + g h D_u \log a' \\ \frac{1}{a'} D_v \log P' = -D_t^2 v - 2 (n \cos \theta + \cos \rho D_t \mu) D_t u - F_v + g h D_v \log a' \end{cases}$$

We have, from the trigonometrical relations of a spherical triangle,—

$$\cos \theta = \cos \psi \cos \rho - \sin \psi \sin \rho \cos \mu$$

Where the range of ρ is so small that we can neglect the second term of this expression in comparison with the first, and put $\cos \rho = 1$ without material error, (22) becomes—

$$(23) \quad \begin{cases} \frac{1}{a'} D_u \log P' = -D_t^2 u + (2 n \cos \psi + D_t \mu) D_t v - F_u + g h D_u \log a' \\ \frac{1}{a'} D_v \log P' = -D_t^2 v - 2 (n \cos \psi + D_t \mu) D_t u - F_v + g h D_v \log a' \end{cases}$$

17. Besides the preceding equations (15) and (23) showing the relations between the motions and pressure of the fluid, there is still another condition, which must always be satisfied in the case of motions of the atmosphere, which is, that the volume of air which occupies any given space must be the same, and the amount of air directly proportional to its density, and the motions of the atmosphere must always be such as to satisfy these conditions. The mathematical expression of this condition in any case where such can be formed, is called the equation of continuity.

CHAPTER II.

THE TEMPERATURE AND PRESSURE OF THE ATMOSPHERE AT THE EARTH'S SURFACE OBTAINED FROM OBSERVATION.

18. In most of the equations of the preceding chapter, there are found the two functions α' and P' , which it is necessary to determine for all parts of the earth's surface from observation. It is seen from (11) that α is a function of t the temperature, α' being the value of α corresponding to the temperature of the atmosphere at the earth's surface. With the value of t , therefore, for all parts of the earth's surface, we obtain from (11) the expression of α' in a function of θ and φ , and consequently of u and v , from which we obtain the functions $D_u \log \alpha'$ and $D_v \log \alpha'$.

The general equations of motion and pressure of the atmosphere contained in the preceding chapter are of such a character, on account of their complexity, and the unknown friction terms entering into them, that they do not admit of a complete solution, and it is therefore important to obtain from observation as many as possible of the functions entering into those equations. If the general equations could be completely solved, we should need only the temperature from observation, and the solution of the equations would then give the atmospheric pressures for all parts of the earth belonging to this temperature, and we should not need the pressures from observation, except for a verification of theory. But as it is, we also need the observed pressures in the different parts of the earth's surface, in order to enable us to obtain other unknown quantities depending upon these pressures, which cannot be obtained from the solution of the equations.

The following tables, I and II, of approximate temperatures for the two extremes, January and July, have been obtained by interpolation from Buchan's Charts of Isothermal Lines, with some corrections first applied, to make them agree with recent observations. The authority for these corrections is derived mostly from the "Contributions to our Knowledge of the Meteorology of Cape Horn and the West Coast of South America," published by the authority of the Meteorological Committee of London. These contributions give temperatures generally from four to six degrees higher for these parts than those indicated by Buchan's isothermal lines for July, and three or four degrees higher than those indicated by his lines for January. As the temperature of the latitude of Cape Horn must be very nearly the same for all longitudes, the temperatures of the extreme southern latitudes in the following tables have been increased accordingly all around the globe. The last column of these tables contain the means of the temperature for all the different longitudes. Although the numbers given for the different longitudes are only approximate, and may in some instances be considerably in error, yet the means of so many longitudes must give very nearly the averages of the different latitudes of the globe, and will be sufficiently accurate for our purpose. And the local variations of temperature independent of latitude will be only needed approximately for explaining in a general way the phenomena depending upon them, and not for any accurate comparisons of theory with observation.

TABLE I.—*Showing the approximate mean temperature in degrees of Fahrenheit for the different parts of the earth's surface in*

JANUARY.

Latitude.	Longitudes west.																		Mean.
	180	170	160	150	140	130	120	110	100	90	80	70	60	50	40	30	20	10	
80	—33	—32	—32	—33	—34	—35	—37	—39	—40	—39	—36	—30	—24	—22	—20	—18	—15	—12	—10
70	—26	—24	—22	—23	—24	—26	—28	—30	—32	—31	—30	—23	—15	—5	4	8	12	18	22
60	10	11	12	12	11	5	—5	—13	—20	—24	—22	—16	0	12	20	28	36	36	34
50	40	40	40	40	40	36	32	22	10	5	0	5	10	26	38	42	44	42	38
40	50	49	50	51	50	50	46	42	36	32	30	32	38	48	54	56	54	52	48
30	60	59	58	57	58	57	56	54	54	54	54	58	62	66	66	66	64	62	60
20	72	71	70	69	68	66	66	66	66	70	72	71	74	74	74	74	74	74	74
10	76	76	76	76	76	76	76	76	76	76	78	78	78	76	76	78	80	82	85
0	80	80	80	80	80	80	80	80	80	82	82	82	82	81	80	80	80	80	80
—10	82	82	81	80	80	80	80	80	80	80	80	82	84	84	84	82	78	78	78
—20	78	78	78	78	78	77	76	76	76	76	78	80	82	82	80	78	76	75	74
—30	74	74	73	72	72	72	71	71	70	69	69	72	74	74	74	74	72	71	70
—40	64	64	64	64	64	62	62	62	62	62	64	66	67	68	68	68	66	62	62
—50	52	52	52	51	51	51	51	51	50	50	55	54	55	55	55	55	55	53	52
—60	39	39	39	38	38	38	38	38	37	37	38	40	41	42	42	42	42	41	39

Latitude.	Longitudes east.																	Mean.
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	
80	—10	—10	—10	—10	—10	—10	—12	—15	—20	—24	—28	—32	—33	—34	—34	—34	—33	—25.0
70	25	18	5	—6	—10	—10	—12	—15	—18	—22	—27	—32	—36	—38	—38	—36	—30	—15.5
60	30	24	16	10	6	0	—4	—8	—12	—16	—24	—30	—30	—24	—16	0	12	1.7
50	34	28	24	18	12	6	4	0	0	0	0	0	4	10	18	24	34	21.3
40	46	44	42	40	35	30	30	30	30	30	26	20	22	30	36	40	44	40.0
30	58	56	55	54	53	52	51	50	47	44	42	40	44	50	53	56	60	55.2
20	74	74	72	68	66	68	70	72	72	72	72	70	70	72	72	72	72	71.0
10	88	90	86	80	80	80	82	82	80	78	76	76	76	76	76	76	76	78.7
0	84	90	90	86	84	82	82	82	81	80	80	80	80	80	80	80	80	81.2
—10	86	90	90	86	84	82	82	82	82	82	82	82	82	82	82	82	82	82.2
—20	82	86	86	84	83	82	82	82	82	82	82	82	82	82	82	80	80	80.0
—30	76	78	78	78	78	78	78	76	74	74	73	72	72	72	72	72	74	73.4
—40	62	64	64	64	64	64	64	64	64	64	64	64	64	64	63	62	63	63.8
—50	52	52	51	51	51	51	51	51	51	51	51	51	51	50	50	50	50	52.0
—60	37	36	35	35	36	36	37	37	38	38	38	38	38	39	39	39	39	38.5

TABLE II.—*Showing the approximate temperature in degrees of Fahrenheit for the different parts of the earth's surface in*

JULY.

Latitude.	Longitudes west.																		Mean.
	180	170	160	150	140	130	120	110	100	90	80	70	60	50	40	30	20	10	
80	34	33	32	32	32	32	32	32	32	33	34	34	34	32	30	30	30	31	32
70	40	40	40	46	50	50	46	44	42	40	38	40	42	42	38	38	38	40	43
60	48	50	52	54	58	62	62	60	56	52	48	45	46	47	48	50	54	55	66
50	60	58	56	56	60	62	68	70	70	68	60	60	60	60	60	62	64	64	64
40	66	66	64	62	60	62	68	74	84	84	76	72	70	70	70	70	70	72	74
30	72	69	66	66	66	68	70	80	88	88	84	82	80	78	77	78	78	81	81
20	80	76	76	76	76	76	76	80	84	85	84	82	82	81	80	82	84	88	90
10	80	80	80	80	80	81	82	82	82	83	84	84	84	82	81	81	82	84	88
0	76	77	78	78	78	78	78	79	80	81	82	82	84	82	78	78	78	78	78
—10	74	74	74	73	75	74	73	74	75	74	75	76	76	75	74	74	74	74	74
—20	66	68	68	68	68	68	68	69	68	67	66	68	70	72	72	70	68	68	68
—30	60	60	60	60	60	60	60	60	60	59	58	58	60	64	64	64	64	66	62
—40	52	51	52	53	54	54	54	53	52	50	50	48	48	49	50	52	54	55	56
—50	45	45	45	45	45	44	43	43	43	43	43	43	43	44	45	45	45	45	45
—60	34	34	33	33	33	32	32	32	31	31	32	33	32	33	34	34	34	33	33

Latitude.	Longitudes east.																	Mean.
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	
80	33	34	34	34	33	32	32	32	33	34	35	36	37	38	37	36	35	34.1
70	46	50	48	44	40	38	40	42	46	50	50	52	52	52	51	50	46	44.3
60	60	62	63	64	64	64	64	65	65	65	65	65	64	60	58	54	52	57.0
50	65	66	68	70	72	74	74	74	74	72	71	70	67	64	63	60	60	65.5
40	76	78	78	78	78	78	80	82	82	82	80	78	74	70	68	66	65	73.0
30	82	83	86	88	90	90	90	89	88	87	85	82	80	78	76	74	71	80.0
20	91	92	92	91	90	90	89	88	87	86	85	83	82	82	81	80	80	84.2
10	90	90	90	88	86	84	82	82	82	83	83	83	82	82	82	82	81	83.2
0	80	84	84	80	78	78	78	78	79	80	81	80	78	76	76	76	76	79.0
—10	78	80	80	78	77	76	76	76	76	76	76	76	75	74	74	74	73	75.2
—20	70	74	74	74	74	74	72	70	69	68	68	68	68	68	68	68	69	69.5
—30	61	60	60	60	60	60	60	60	60	60	59	58	56	54	56	58	59	60.1
—40	55	54	54	54	53	52	52	52	52	52	51	50	50	50	51	52	52	52.0
—50	45	45	45	45	44	43	43	43	42	41	41	41	41	41	41	42	43	43.5
—60	32	31	31	30	30	30	31	31	32	32	32	32	32	32	32	31	32	32.0

Tables III and IV give the approximate mean annual temperatures for each tenth degree of latitude and longitude and the approximate mean annual range of temperature, and likewise the means of all the longitudes. The former have been obtained by taking simply the mean of the extreme mean temperatures of January and July, and the latter by taking the differences of these extremes. Any one must be struck, from an inspection of Table IV, with the great differences between the mean annual ranges of temperature of the northern and southern hemispheres, arising from the unequal distribution of land and water in the two hemispheres.

TABLE III.—*Showing the approximate mean annual temperature in degrees of Fahrenheit for the different parts of the earth's surface.*

$\frac{1}{2}$ (January + July).

Latitude.	Longitudes west.																		Mean.
	180	170	160	150	140	130	120	110	100	90	80	70	60	50	40	30	20	10	
80	0	0	0	0	- 1	- 1	- 2	- 3	- 4	- 3	- 1	2	5	5	5	6	8	10	11
70	7	8	9	11	13	12	9	7	5	4	4	8	13	19	21	23	25	29	32
60	29	30	32	33	35	34	29	24	18	14	13	14	23	29	34	39	45	45	45
50	50	49	48	48	50	49	47	46	40	36	30	32	35	43	49	52	54	53	51
40	53	57	57	56	56	57	57	58	60	58	53	52	54	59	62	63	62	62	56
30	66	64	62	61	61	61	62	67	71	71	69	70	71	72	72	72	71	72	71
20	76	74	73	72	72	71	71	73	75	82	78	77	78	77	77	78	79	81	82
10	78	78	78	78	78	78	79	79	79	80	81	81	81	80	80	80	81	83	86
0	78	78	79	79	79	79	79	80	81	81	82	82	83	82	79	79	79	79	79
-10	78	78	78	77	77	77	77	77	77	77	78	79	80	80	79	78	76	76	76
-20	73	73	73	73	73	73	72	72	72	72	72	74	76	77	76	74	72	72	71
-30	67	67	66	66	66	66	65	65	65	64	64	65	67	69	69	69	69	69	66
-40	58	58	58	58	59	58	58	58	57	56	56	56	57	58	59	60	60	59	59
-50	48	48	48	48	48	48	47	47	47	47	49	49	49	49	50	50	50	50	48
-60	36	36	36	36	35	35	35	35	34	34	35	37	37	37	38	38	38	37	36

Latitude.	Longitudes east.																	Mean.
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	
80	11	12	12	12	12	11	10	8	6	5	4	2	2	2	2	1	1	4.5
70	35	34	27	19	15	14	14	13	14	14	12	10	8	7	6	7	8	14.4
60	45	43	40	37	35	32	30	28	26	25	21	17	17	18	21	27	32	29.3
50	50	47	46	44	42	40	39	37	37	36	35	35	35	37	40	42	47	43.4
40	61	61	60	59	57	54	55	56	56	56	53	49	48	50	52	53	54	56.5
30	70	70	70	71	71	71	71	70	68	66	64	61	62	64	65	65	65	67.6
20	82	83	82	80	78	79	79	80	80	80	79	78	76	77	77	76	76	77.6
10	89	90	88	84	83	82	82	82	81	80	80	80	79	79	79	79	78	81.0
0	82	87	87	83	81	80	80	80	80	80	80	80	79	78	78	78	78	80.1
-10	82	85	85	82	80	79	79	79	79	79	79	79	79	78	78	78	77	78.7
-20	76	80	80	79	79	78	77	76	75	75	75	75	75	75	75	75	74	74.7
-30	68	69	69	69	69	69	69	68	67	67	66	65	64	63	64	65	66	66.7
-40	58	59	59	59	59	58	58	58	58	58	58	57	57	57	57	57	57	57.9
-50	48	48	48	48	48	47	47	47	47	46	46	46	46	46	46	46	47	47.8
-60	35	33	33	33	33	34	33	33	33	34	35	35	35	34	34	34	35	35.3

TABLE IV.—*Showing the approximate mean annual range of temperature in degrees of Fahrenheit for the different parts of the earth's surface.*

(July — January.)

Latitude.	Longitudes west.																		Mean.
	180	170	160	150	140	130	120	110	100	90	80	70	60	50	40	30	20	10	0
80	67	65	64	65	66	67	69	71	72	72	70	64	58	54	50	48	45	43	42
70	66	64	62	69	74	76	74	74	74	71	68	63	57	47	34	30	26	22	21
60	38	40	40	42	47	57	67	73	76	76	70	61	46	35	28	22	18	19	22
50	20	18	16	16	20	26	36	48	60	63	60	55	50	34	22	20	20	22	26
40	16	17	14	11	8	10	22	32	48	52	46	40	32	22	16	14	16	20	26
30	12	16	8	9	10	13	16	26	34	34	30	24	18	12	11	12	14	19	21
20	8	5	6	7	8	9	10	15	18	15	12	11	8	7	6	8	10	14	16
10	4	4	4	4	4	5	6	6	6	7	6	6	6	6	5	3	2	2	3
0	-4	-3	-2	-2	-2	-2	-2	-1	0	-1	0	0	2	1	-2	-2	-2	-2	-2
-10	-8	-8	-7	-7	-5	-6	-7	-6	-5	-6	-5	-6	-8	-9	-10	-8	-4	-4	-4
-20	-10	-10	-10	-10	-10	-9	-8	-7	-8	-9	-12	-12	-12	-16	-8	-8	-8	-7	-6
-30	-14	-14	-13	-12	-12	-12	-11	-11	-10	-10	-11	-14	-14	-16	-10	-10	-8	-5	-8
-40	-12	-13	-12	-11	-10	-8	-8	-9	-10	-12	-12	-16	-18	-18	-18	-16	-12	-7	-6
-50	-7	-6	-6	-6	-6	-7	-8	-9	-10	-11	-12	-11	-11	-12	-11	-10	-10	-8	-7
-60	-5	-5	-5	-5	-5	-6	-6	-6	-6	-6	-6	-7	-7	-9	-8	-8	-8	-8	-6

Latitude.	Longitudes east.																	Mean.
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	
80	43	44	44	44	43	42	44	47	53	58	63	68	70	72	71	70	68	59.1
70	21	32	43	50	50	48	52	57	64	72	77	84	88	90	89	86	76	59.8
60	30	38	47	54	58	64	68	73	77	81	89	95	94	84	74	54	40	55.3
50	31	38	44	52	66	68	70	74	74	72	71	70	63	54	45	36	26	44.2
40	30	34	36	38	43	48	50	52	52	52	54	58	52	40	32	26	21	33.0
30	24	27	31	34	37	38	39	39	41	43	43	42	36	28	23	18	11	24.8
20	17	18	20	23	24	22	19	16	15	15	14	15	13	16	9	8	8	13.2
10	2	6	4	8	6	4	0	0	2	5	7	7	6	6	6	6	5	4.5
0	-4	-6	-6	-6	-6	-4	-4	-4	-2	0	1	0	-2	-4	-4	-4	-4	-2.2
-10	-8	-10	-10	-8	-7	-6	-6	-6	-6	-6	-6	-6	-7	-8	-8	-8	-9	-7.0
-20	-12	-12	-12	-10	-9	-8	-10	-12	-13	-14	-14	-14	-14	-14	-14	-12	-11	-10.5
-30	-15	-13	-18	-18	-18	-18	-18	-16	-14	-14	-14	-14	-16	-18	-16	-14	-15	-13.3
-40	-7	-10	-10	-10	-11	-12	-12	-12	-12	-12	-13	-14	-14	-14	-12	-10	-11	-11.8
-50	-7	-7	-6	-6	-7	-8	-8	-8	-9	-10	-10	-10	-10	-9	-9	-8	-7	-8.5
-60	-5	-5	-5	-5	-6	-6	-6	-6	-6	-6	-6	-6	-6	-7	-7	-8	-7	-6.5

The numbers in Tables I and II being averages for the months of January and July, and not the extremes of the average mean daily temperature, the results of this table require a small correction, which is the difference between the averages of the month and the extreme of the average mean daily temperatures, in order to obtain the absolute mean range of temperature; but this correction is quite small.

19. By reducing the mean temperature in the last columns of Tables I, II, and III to centigrade degrees, we get the second, third, and fourth columns in—

TABLE V.

θ	Temperature.				Residuals.
	January.	July.	Mean.		
			Observed.	Computed.	
°	°	°	°	°	°
0	-----	-----	-----	-17.0	-----
10	-31.9	1.0	-15.5	15.8	+0.3
20	26.5	6.9	9.8	10.2	+0.4
30	16.9	13.8	-1.6	-2.2	+0.6
40	-6.0	18.6	+6.3	+6.5	-0.2
50	+4.5	22.8	13.6	14.4	-0.8
60	12.9	26.6	19.8	20.4	-0.6
70	21.7	29.0	25.3	24.3	+1.0
80	25.9	28.4	27.2	26.4	+0.8
90	27.3	26.1	26.7	26.8	-0.1
100	27.9	24.0	25.9	26.0	-0.1
110	26.6	20.8	23.7	23.8	-0.1
120	23.0	15.6	19.3	20.2	-0.9
130	17.6	11.1	14.4	14.9	-0.5
140	11.1	6.4	8.8	8.2	+0.6
150	+3.6	0.0	+1.8	+0.9	+0.9
160	-----	-----	-----	-5.8	-----
170	-----	-----	-----	10.6	-----
180	-----	-----	-----	-12.4	-----

If we put for the mean annual temperature—

$$(24) \quad t = t_0 + a \cos \theta + b \cos 2 \theta + c \cos 3 \theta + d \cos 4 \theta$$

we get from this equation, with the fifteen observed values of t , and the corresponding values of θ in the first column, fifteen equations of condition for determining, by the method of least squares, the values of t_0 , a , b , c , and d . With the values so determined, we get—

$$(25) \quad . . \quad t = 8^{\circ}.50 - 1^{\circ}.75 \cos \theta - 20^{\circ}.95 \cos 2 \theta - 1^{\circ}.00 \cos 3 \theta - 2^{\circ}.66 \cos 4 \theta$$

From this expression of t we get the computed values of t in the third column of the preceding table, which satisfies the observations with the residuals contained in the last column. Although this expression may represent the observations best for the latitudes for which they have been made, yet it cannot be regarded as representing the temperatures very accurately at or near the poles, especially the south pole.

20. In order to obtain the mean temperature of the earth's surface, we must integrate the expression obtained from (24) by multiplying it into $\sin \theta$ and integrating with regard to θ , by which we get—

$$\begin{aligned}
 \int_{\theta} t \sin \theta &= \int_{\theta} \sin \theta \left(t_0 + a \cos \theta + b \cos 2 \theta + c \cos 3 \theta + d \cos 4 \theta \right) \\
 &= - \left(t_0 - \frac{1}{2} b \right) \cos \theta - \frac{1}{4} (a - c) \cos 2 \theta - \frac{1}{6} (b - d) \cos 3 \theta - \frac{1}{8} c \cos 4 \theta \\
 &\quad - \frac{1}{10} d \cos 5 \theta + C
 \end{aligned}$$

in which—

$$C = \left(t_0 - \frac{1}{2}b\right) + \frac{1}{4}(a-c) + \frac{1}{6}(b-d) + \frac{1}{8}c + \frac{1}{10}d = t_0 + \frac{1}{4}a - \frac{1}{3}b - \frac{3}{8}c - \frac{1}{15}d$$

This integral gives for the northern hemisphere, using the values of t_0 , a , b , c , and d in (25),—

$$\int_{\theta} t \sin \theta = t_0 + \frac{1}{2}a - \frac{1}{3}b - \frac{1}{2}c - \frac{1}{15}d = 15^{\circ}.30$$

and for the southern hemisphere,—

$$\int_{\theta} t \sin \theta = t_0 - \frac{1}{2}a - \frac{1}{3}b + \frac{1}{2}c - \frac{1}{15}d = 16^{\circ}.05$$

The mean of these two results gives $15^{\circ}.67$ for the mean temperature of the whole surface of the earth.

From Dove's Charts of Isothermal Lines, which do not extend beyond the middle latitudes in the southern hemisphere, it has been inferred that the southern hemisphere is colder than the northern, and this has been the accepted view ever since his charts were first published, in the year 1852; but, from the results obtained above, it is seen that the mean temperature of the southern hemisphere is the greater of the two. If, however, we compare the values of t in the preceding table for the two hemispheres between the parallels of 30° north and south, we find that the southern hemisphere is the colder of the two between these parallels; but beyond the parallels of about 35° the temperatures of the southern hemisphere become greater than those of the corresponding latitudes of the northern hemisphere, so that the average temperature is also greater, as shown above. The cause of this is found in the unequal distribution of land and water in the two hemispheres; for we now know, both from theory and observation, that there is a constant, though very slow, interchange of the water of the ocean between the equatorial and polar regions, which tends to diminish in some measure the difference of temperature between these regions, so that in the southern hemisphere, where there is mostly water, the temperatures of the higher latitudes must be greater than those of the same latitudes in the northern hemisphere, and the reverse for the lower latitudes. The small differences above between the mean temperatures of the two hemispheres is perhaps only of the order of the possible errors of these results, so that we cannot infer that there is any real difference in the averages of the two hemispheres.*

21. We now come to the subject of atmospheric pressure on the different parts of the earth's surface, upon which the values of $D_n P'$ and $D_s P'$ in the general equations of the preceding chapter depend. This pressure cannot be determined from theory, on account of the complexity of the equations and the uncertain element of friction entering into them, and we shall, therefore, endeavor to determine it, so far as possible, from observation. The very valuable and exhaustive paper by Buchan on this subject, the "Mean Pressure of the Atmosphere and the Prevailing Winds over the Globe, for the Month and for the Year," published in the year 1869,† left nothing undone which could have been done at the time in the way of determining the atmospheric pressure from observation on the different parts of the globe; but since that time there has been so great an accumulation of barometrical observations from almost all parts of the world, that it does not seem proper, in our present researches on the subject, not to avail ourselves of these observations, at least in some measure, in determining this pressure still more accurately for all places from which additional and more recent observations have been obtained. It is also thought that the knowledge which we now have of the relations between the barometric pressures and the velocities and directions of the winds, obtained from theory and corroborated by observation, can be now used in laying down isobaric lines for those countries and the vast expanse of ocean from which we have none, or at least only a very few, observations, much more accurately than has been done heretofore, since much

* See two papers on this subject by Dr. Hann in the "Zeitschrift der österreichischen Gesellschaft für Meteorologie," Band vii, S. 261, and Band xii, S. 100, which were not seen until after the preceding results were obtained, and in which these results are corroborated. If, however, the observations upon which the results obtained by Dr. Hann are based had been on hand at the time, the values of t in Table V would most probably have been diminished a very little in the extreme southern latitudes, and then the results obtained for the average temperatures of the two hemispheres might have been about equal.

† Transactions of the Royal Society of Edinburgh, vol. xxv.

may be inferred with regard to the barometric pressures from the known forces and directions of the winds. We shall, therefore, undertake to make out new charts of isobaric lines, not for each month, but simply for the mean annual pressures of the globe, and for the two extremes of January and July in the northern hemisphere, using for this purpose all the observations on hand, except in some parts of Europe, where there is so great an accumulation of them that all were not considered necessary for our purpose, since, in a general system of isobars for the whole globe, in which, from the scarcity of observations in many parts, there is much that is necessarily hypothetical and conjectural, it is not worth while to aim at extreme accuracy in certain comparatively very small parts of the surface of the globe. We shall also make out charts showing the annual variation of the atmospheric pressures over all parts of the globe, so far as this can be determined from observation, so that, with the mean annual pressure and the annual variation, the mean monthly pressures may be readily obtained, even with greater accuracy than they can be obtained from charts laid down from monthly averages, for it will be shown that all other neglected inequalities are generally either insensible, or at least smaller than the probable errors of monthly averages. These charts will be given on polar projections of the two hemispheres, since each hemisphere contains a complete system of winds and barometric pressures which are similar, and because in such projections there is less distortion of the parts in the higher latitudes, where, in the northern hemisphere, there are some features in the winds and barometric pressures, and their annual variations, which it is desirable to have more accurately represented than they can be in the usual projections.

22. For our purpose, we have put for each station for which we have observations of the pressure P—

$$(26) \quad P = B_0 + B_1 \cos(\varphi - \varepsilon_1) + B_2 \cos(2\varphi - \varepsilon_2) + \&c. \\ = B_0 + M_1 \cos \varphi + N_1 \sin \varphi + M_2 \cos 2\varphi + N_2 \sin 2\varphi + \&c.$$

in which—

$$M_1 = B_1 \cos \varepsilon_1; \quad N_1 = B_1 \sin \varepsilon_1$$

$$M_2 = B_2 \cos \varepsilon_2; \quad N_2 = B_2 \sin \varepsilon_2$$

$$\tan \varepsilon_1 = \frac{N_1}{M_1}; \quad \tan \varepsilon_2 = \frac{N_2}{M_2}$$

$$B_1 = \frac{M_1}{\cos \varepsilon_1} \text{ or } \frac{N_1}{\sin \varepsilon_1}; \quad B_2 = \frac{M_2}{\cos \varepsilon_2} \text{ or } \frac{N_2}{\sin \varepsilon_2}$$

In the preceding expression of P —

B_0 = the mean annual barometric pressure ;

B_1 = the coefficient of the annual inequality ;

B_2 = that of the semi-annual inequality ;

φ = an angle increasing in proportion to the time at a rate, on the average, of 30° per month ;

ε_1 = a constant which is the value of φ at the time of the maximum of the annual inequality ; and

ε_2 = a constant which is the value of 2φ at the time of the maximum of the semi-annual inequality.

If we put S_n for the monthly mean of the barometer for the n th month and the epoch of φ in the middle of December, we have, by regarding each month as the twelfth part of a year,—

$$B_0 = \frac{\sum_n S_n}{12}$$

$$6 M_1 = \begin{pmatrix} S_1 - S_7 \\ S_{11} - S_5 \end{pmatrix} \cos 30^\circ + \begin{pmatrix} S_2 - S_8 \\ S_{10} - S_4 \end{pmatrix} \cos 60^\circ + \begin{pmatrix} S_{12} - S_6 \end{pmatrix}$$

$$6 N_1 = \begin{pmatrix} S_1 - S_7 \\ S_5 - S_{11} \end{pmatrix} \sin 30^\circ + \begin{pmatrix} S_2 - S_8 \\ S_4 - S_{10} \end{pmatrix} \sin 60^\circ + \begin{pmatrix} S_3 - S_9 \end{pmatrix}$$

$$6 M_2 = \begin{pmatrix} S_1 - S_4 \\ S_5 - S_8 \\ S_7 - S_{10} \\ S_{11} - S_2 \end{pmatrix} \cos 60^\circ + \begin{pmatrix} S_{12} - S_3 \\ S_6 - S_9 \end{pmatrix}; \quad 6 N_2 = \begin{pmatrix} S_1 - S_4 \\ S_2 - S_5 \\ S_7 - S_{10} \\ S_8 - S_{11} \end{pmatrix} \sin 60^\circ$$

With the values of M_1 , M_2 , N_1 , and N_2 , given by these formulæ, we get from the preceding expressions the values of B_1 , B_2 , ϵ_1 , and ϵ_2 .

The preceding expression of P is simply a transformation of what is usually called Bessel's formula, in which sines are used, instead of the cosines in the form of expression here adopted. This form is preferable to the other, since from the values of ϵ_1 and ϵ_2 the times of the maxima of the inequalities are more readily obtained, and this is made still more convenient by having the constants ϵ_1 and ϵ_2 of the angles in the expression negative, as is usual in all tidal expressions of this sort, for in this case we have merely to add to the assumed epoch of the time-angles the time which is required for the angles to change by the quantity ϵ_1 or ϵ_2 in order to have the time of the maximum of the inequality. For the annual inequality, we divide ϵ_1 , expressed in degrees, by 30, and for the semi-annual inequality ϵ_2 by 60, in order to get the time in months by which the maximum follows the assumed epoch of the middle of December.

The preceding formulæ for obtaining M_1 , M_2 , N_1 , and N_2 follow directly from the determination of these quantities by the method of least squares in the special case of twelve equal divisions of the period of the principal inequality. The values, therefore, of the constants B_1 , B_2 , ϵ_1 , and ϵ_2 thus obtained are the *most probable* values of these constants.

The form of (26) can likewise be applied to represent the temperature of the earth or atmosphere at any place, in which case the principal inequality has an annual period. In the case of both atmospheric pressure and temperature, more than two inequalities need not be considered; for if those of a lower order have sensible coefficients, they are of an order much smaller than the probable errors of the constants determined from observation, unless the period of observation embraces a very long series of years.

23. More convenient formulæ for the determination of the constants in (26) may be obtained upon the principle of averages, and the results obtained upon this principle, though differing a little from the most probable results given by the preceding formulæ, are sufficiently accurate in all meteorological researches, and the differences between the results of the two sets of formulæ will generally be found to be much less than the probable errors of the determinations.

If we put—

a = the average of all the observations of P within the limits of $\varphi = \varphi_I$ and $\varphi = \varphi_{II}$

φ' = the mean value of φ between the limit φ' and φ'' ,

by considering only the first two inequalities in P (26), and supposing the observation t equally distributed in time, we have—

$$(27) \quad a = B_0 + k B_1 \cos (\varphi' - \epsilon_1) + k' B_2 \cos (2 \varphi' - \epsilon_2)$$

in which—

$$(28) \quad \left\{ \begin{aligned} k &= \frac{2 \sin \frac{1}{2} (\varphi_{II} - \varphi_I)}{\varphi_{II} - \varphi_I} = \frac{2 \sin \frac{1}{2} c}{2 n \pi + c} \\ k' &= \frac{\sin (\varphi_{II} - \varphi_I)}{\varphi_{II} - \varphi_I} = \frac{\sin c}{2 n \pi + c} \end{aligned} \right.$$

c in the last form of expression of k and k' being the excess of $\varphi_{II} - \varphi_I$ over an equal number of periods $2 n \pi$ of the inequality.*

Where $\varphi_{II} - \varphi_I = 2 n \pi$, that is, where any number of even multiples of the period of the angle φ is used in taking the average, a , the arc c , and consequently k and k' , vanish, and we have $a = B_0$. The same is sensibly true in the case of a long series of observations, since c , and consequently $2 \sin \frac{1}{2} c$ and $\sin c$, then becomes very small in comparison with $2 n \pi$ in the denominator, in which n denotes the number of periods of the inequality used.

If we take the observations within the limits of the half-period $\varphi_{II} - \varphi_I = \pi$, since $\sin \pi = 0$, we then have $k' = 0$, and (27) becomes—

$$a = B_0 + k B_1 \cos (\varphi' - \epsilon_1)$$

* The preceding results of this section are demonstrated in my Tidal Researches, p. 158, published as an appendix to the United States Coast Survey Report for 1874.

The same is of course true if we take any number of successive half-periods within the same limits of φ . If we take the average for the other alternate half-periods, we get—

$$a = B_0 - k B_1 \cos (\varphi' - \epsilon_1)$$

If we therefore reverse the signs of the observations for the latter, we get for the average—

$$(29) \quad \begin{aligned} a &= k B_1 \cos (\varphi' - \epsilon_1) \\ &= k M_1 \cos \varphi' + k N_1 \sin \varphi' \end{aligned}$$

If we put S_n = the monthly average of the n th month, and assume the epoch of the angle φ at the beginning of January, and take the limits φ_i and φ_{ii} so that $\varphi' = 0$, and put—

$$(30) \quad \begin{cases} A = S_1 + S_2 + S_3 \\ B = S_4 + S_5 + S_6 \\ C = S_7 + S_8 + S_9 \\ D = S_{10} + S_{11} + S_{12} \end{cases}$$

we get from (29), since in this case $\cos \varphi' = 1$ and $\sin \varphi' = 0$,—

$$12 a = 12 k M_1 = S_{10} + S_{11} + S_{12} + S_1 + S_2 + S_3 - S_4 - S_5 - S_6 - S_7 - S_8 - S_9 = (A + D) - (B + C)$$

If we now assume the limits so that φ' falls on the 1st of April, we have in (29) $\sin \varphi' = 1$, and $\cos \varphi' = 0$, and we get—

$$12 a = 12 k N_1 = S_1 + S_2 + S_3 + S_4 + S_5 + S_6 - S_7 - S_8 - S_9 - S_{10} - S_{11} - S_{12} = (A + B) - (C + D)$$

With the value of $k = \frac{2}{\pi}$ from (28), since $\varphi_{ii} - \varphi_i$ in this case is equal to $\frac{1}{2} \pi$, these give—

$$(31) \quad \begin{cases} M_1 = 0.1309 (A + D - B - C) \\ N_1 = 0.1309 (A + B - C - D) \end{cases}$$

With the values of M_1 and N_1 , we obtain B_1 and ϵ_1 by the formulæ in § 22.

If we now take the averages for intervals of the angle, or $\varphi_{ii} - \varphi_i$ equal to $\frac{1}{2} \pi$, or 90° , changing the signs of the observations for each second alternating interval, it is readily seen that the first inequality in (27) is eliminated, since in this case (28) gives $k = 0$, and we get, instead of (29),—

$$(32) \quad a = k' M_2 \cos 2 \varphi' + k' N_2 \sin 2 \varphi'$$

In this case, the half-period of the angle embraces only three months; and, in order to obtain the value of M_2 , we must use intervals of two months only, so as to have φ' fall in the middle of the interval, and we thus get, since $\cos 2 \varphi' = 1$ and $\sin 2 \varphi' = 0$,—

$$8 a = 8 k' M_2 = S_{12} + S_1 - S_3 - S_4 + S_6 + S_7 - S_9 - S_{10}$$

In this case, we have the range between the limits $\varphi_{ii} - \varphi_i = 60^\circ$, and hence we get from (28),—

$$k' = \frac{\sin 60^\circ}{\frac{1}{3} \pi} = 0.827$$

If we now take the limits of the alternate intervals φ_i and φ_{ii} so that φ' of the first falls on the middle of February, we shall have $\sin 2 \varphi' = 1$ in (32) and $\cos 2 \varphi' = 0$, and consequently—

$$12 a = 12 k' N_2 = S_1 + S_2 + S_3 - S_4 - S_5 - S_6 + S_7 + S_8 + S_9 - S_{10} - S_{11} - S_{12}$$

Since we have in this case $\varphi_{ii} - \varphi_i = 90^\circ$, (28) gives—

$$k' = \frac{1}{\frac{1}{2} \pi} = \frac{2}{\pi}$$

We therefore get from the preceding expressions of $8 k' M_2$ and $12 k' N_2$, with the corresponding values of k' for each,—

$$(33) \quad \begin{cases} M_2 = 0.1511 [(S_1 - S_3) + (S_6 - S_4) + (S_7 - S_9) + (S_{12} - S_{10})] \\ N_2 = 0.1309 [(A + C) - (B + D)] \end{cases}$$

With the values of M_2 and N_2 given by (33), we get, by the formulæ in § 26, the values of B_2 and ε_2 .

It is very convenient in practice to obtain by (31) and (33) the values of M_1 , N_1 , and M_2 , N_2 , from observation, and the values so found are very accurate when the expression of P (26) is so convergent as to make the inequalities after the first two very small or entirely insensible, as is the case in meteorology. In a comparison of the values of B_1 for thirty cases, obtained from series of meteorological observations by both of the methods which have been given, the average of the differences by the two methods, taken without regard to signs, was only 0.13^{mm} , the maximum being 0.39^{mm} , and in the case of the values of B_2 the differences were of the same order. Even in cases in which it is desirable to use the former more accurate formulæ of the preceding section, these latter will be found a very convenient check within very narrow limits.

Where the range of angle $\varphi'' - \varphi'$, belonging to a group of observations of which the average a is taken, is small, the values of k and k' (28) do not differ sensibly from unity, and, by comparing (26) with (27), it is seen that the average can be used as the value of the function belonging to the middle of the interval, in which the value of φ is φ' . If, however, the range should be considerable, this average must be corrected in order to obtain the value of the function for the middle of the group of observations. From (26) and (27) we get—

$$(34) \quad P = a + (1-k) B_1 \cos (\varphi' - \varepsilon_1) + (1-k') B_2 \cos (2\varphi' - \varepsilon_2)$$

The last two terms, therefore, will be small corrections to the average a in order to get the value of the function P for the middle of the group, for which $\varphi = \varphi'$. For instance, if we had monthly averages, we should have $\varphi'' - \varphi' = 30^\circ$, and (28) would give $1 - k = 1 - 0.9886 = 0.0114$, and $1 - k' = 1 - 0.9549 = 0.0451$, and the correction is—

$$P - a = 0.0114 B_1 \cos (\varphi - \varepsilon_1) + 0.0451 B_2 \cos (2\varphi - \varepsilon_2)$$

The last term, on account of the smallness of B_2 , is generally very small in monthly averages. When the range $\varphi'' - \varphi'$ is larger, of course these corrections become of more importance.

24. The constants in the last column of the following table have been deduced, by means of the preceding formulæ, from the monthly means reduced to sea-level and the gravity of the parallel of 45° , contained in Rikatcheff's paper entitled "La Distribution de la Pression Atmosphérique dans la Russie d'Europe," published in the *Repertorium für Meteorologie*, Band iv, Heft 1, S. 46, 47.

TABLE VI.

Place.	Latitude.	Longitude.	Altitude.	Years of observation.	B ₀ .	B ₁ .	B ₂ .	e ₁ .	e ₂ .
	° ' "	° ' "	m.		mm.	mm.	mm.	°	°
Kem	64 57	34 30	13.7	7	757.3	1.7	1.1	123	22
Archangel	64 33	40 32	10.7	19	757.5	0.7	0.5	151	219
Helsingfors	60 10	24 57	11.6	15	759.6	0.5	0.5	86	192
St. Petersburg	59 56	30 16	4.5	50	759.9	0.6	0.5	35	197
Boyalovsk	59 46	60 1	193.7?	16	761.6	3.6	0.5	19	187
Reval	59 26	24 45	(?)	17	759.7	0.3	1.3	143	248
Port Baltic	59 21	24 3	8.5	33	759.8	0.2	0.5	145	260
Dorpat	58 23	26 43	68.3 & 43.0?	22	760.3	0.2	0.3	225	180
Nijno Taguilak	57 56	60 5	189.4?	26	762.3	4.4	1.7	11	152
Kostroma	57 47	40 55	106.2?	21	761.4	2.3	1.0	11	185
Riga	56 58	24 6	16.6?	21	760.5	0.5	0.4	6	236
Catherineburg	56 49	60 35	283.8	31	762.8	4.5	0.9	14	190
Mitan	56 39	23 44	4.9	19	760.5	0.8	0.6	354	206
Kazan	55 47	49 8	71.1	17	762.4	4.2	1.1	20	193
Moscow	55 46	37 40	145.1	48	762.3	2.3	0.8	359	162
Slatovsk	55 10	59 40	415.4?	33	763.0	5.4	0.2	10	21
Kalouga	54 30	36 15	162.6?	13	762.3	2.7	0.8	354	221
Orel	52 57	36 5	165.2?	8	762.3	2.9	1.4	349	190
Tambov	52 44	41 28	165.5?	14	762.9	3.2	0.3	12	212
Warsaw	52 13	21 2	119.4	35	761.3	1.4	0.2	346	106
Orenburg	51 46	55 6	103.5	28	763.8	5.9	0.7	8	182
Konrsk	51 45	36 8	209.4?	25	762.6	2.8	0.7	350	189
Ouralek Fortress	48 37	61 16	108.9?	8	762.9	7.6	1.6	6	192
Lougan	48 35	39 20	62.0?	31	762.9	4.0	0.5	0	226
Nikolaief	46 58	31 58	55.0 & 19.0	26	762.4	3.1	0.1	356	259
Odessa	46 29	30 44	53.3 & 65.3	18	762.4	2.9	0.3	352	248
Astrakhan	46 21	48 2	-----	12	763.0	4.6	0.5	2	304
Stavropol	45 3	41 59	555.7	27	762.3	4.4	0.5	10	345
Tiflis	41 43	44 47	459.9 & 499.3	26	763.2	4.0	0.9	0	310
Bakou	40 22	49 50	— 16.0	18	761.6	4.4	0.8	2	266
Hammerfest	70 40	23 46	6.4	13	756.4	3.9	1.1	177	262
Happaranda	65 51	24 11	11.6	12	758.5	1.0	0.5	126	224
Stykisholm	65 4	— 22 43	11.3	23	754.0	4.5	2.0	174	222
Reykjavik	64 40	— 22 0	11.0	17	752.7	5.3	0.7	167	283
Christian Sund	63 7	7 45	19.8	8	756.7	2.9	1.3	168	253
Heernösand	62 38	17 57	19.2	12	758.7	2.6	0.9	135	272
Christiania	59 55	10 44	22.7	35	759.0	0.5	0.6	105	220
Upsala	59 52	17 38	24.0	18	758.3	0.3	0.5	124	278
Copenhagen	55 41	12 35	3.6	11	760.5	0.4	1.0	238	284
Stralsund	54 29	12 20	14.9	22	761.0	0.7	0.4	324	267
Stettin	53 35	14 34	21.8	16	761.2	0.7	0.3	297	11
Götersloh	51 54	8 23	81.2	22	761.2	0.3	0.4	243	54
Dublin	53 22	— 6 21	49.8	33	760.3	1.2	0.3	170	354
Berlio	52 30	13 23	46.8	32	761.9	0.3	0.3	305	59
Utrecht	52 5	5 7	13.4	20	761.9	0.3	0.5	241	3
London	51 28	0 0	0.0	-----	761.2	0.7	0.5	155	57
Breslau	51 7	17 2	147.5	39	762.7	1.7	0.5	342	25
Brussels	50 51	4 22	56.7	35	761.6	0.4	0.7	262	0
Prague	50 5	14 23	201.2	19	762.3	1.6	0.8	335	17
Cracow	50 4	19 57	215.7	12	762.3	1.7	0.4	326	58
Breszew	50 3	22 0	213.7	11	762.4	1.7	0.1	328	148
Paris	48 50	2 20	65.8	15	762.4	0.3	1.1	4	26
Vienna	46 55	16 20	191.3	17	762.1	1.7	0.8	335	36
Triest	45 39	13 44	-----	-----	761.1	0.9	0.5	341	47
Bordeaux	44 50	— 0 35	22.9	10	763.1	0.8	0.8	297	48
Orange	44 8	4 48	45.4	36	762.1	0.9	0.5	311	36
Toulouse	43 37	— 1 28	198.1	22	763.2	1.0	1.1	336	40
Constantinople	41 0	28 59	(?)	28	763.0	2.6	0.6	355	15
Naples	40 52	14 15	146.9	11	763.9	0.8	1.5	269	27

TABLE VI—Continued.

Place.	Latitude.	Longitude.	Altitude.	Years of observation.	B ₀ .	B ₁ .	B ₂ .	ε ₁ .	ε ₂ .
	° /	° /	m.		mm.	mm.	mm.	°	°
Lisbon	38 43	— 9 8	135.4	11	760.8	0.8	0.4	7	245
Athens	37 58	23 43	80.5	11	760.3	1.8	0.6	356	247
Alger	36 43	3 3	20.1	13	763.0	1.1	1.2	347	24
Oran	35 40	— 0 38	50.0	3	763.2	1.8	0.9	15	8

25. The results contained in the following table have been deduced in the same manner as those of the preceding table from the monthly means of observations given from time to time in the "Zeitschrift der österreichischen Gesellschaft für Meteorologie." These observations have been, in most cases, collected, revised, and discussed, and, in many cases, corrected, by Dr. Hann, who has spent much time and labor upon them, and to whom, therefore, much credit is due, and the references in the following table are to the "Zeitschrift für Meteorologie" alone, where references will be found to the original sources from which the observations have been obtained, and due credit given. These observations include many of those used by Buchan, and also most of the observations made in all parts of the world since the publication of Buchan's paper, already referred to. These observations as given were not reduced to sea-level, nor to the gravity of the parallel of 45°. For the former reductions, we have used mostly the table on page 91 of Guyot's *Hypsometrical Tables*; but for all places of considerable altitude, the reductions have been made by the following formula:—

$$(35) \quad \log P' - \log P = \frac{h}{18428 (1 + 0.004 t + 0.00001 h)}$$

in which the coefficient of the temperature t is taken so as to include approximately the effect of moisture, and in which the temperature is supposed to decrease 0°.5 for each 100 meters of altitude. The altitude h in the formula must be expressed in meters.

In a few cases, the altitudes were not given, and, where given, there was some uncertainty with regard to the manner in which they may have been obtained. If obtained barometrically, as is sometimes the case with altitudes thus given, then the reduced pressures to sea-level are simply the assumed pressures at sea-level used in determining the altitudes, and hence are worthless. In drawing isobars on the charts, however, those results obtained from observations at considerable altitudes, in which there was considerable uncertainty in the reductions, were not allowed much, if any, weight.

Instead of reducing the monthly means to sea-level, this reduction was applied merely to the mean of the barometric pressures B_0 given in the following table, using the mean temperature. This leaves an inequality in the reduction to sea-level, depending upon the annual inequality of temperature, uncorrected, which requires a correction to be applied to the coefficient B_1 . The formula for this correction, as deduced from (35), is—

$$(36) \quad \Delta P = \Delta B_1 = \frac{h}{18428} \times 0.004 \Delta t \times \frac{P}{0.4343} = \frac{P h \Delta t}{2001280}$$

in which Δt is the variation of the extreme of the mean monthly temperatures in January or July from the mean temperature, and in which the value of P at the height of $\frac{1}{2} h$ should be used when h is great. The corrections given by this formula under the head ΔB_1 in the following table must be applied to B_1 , in order to have the coefficient of annual inequality at the sea-level.

TABLE VII.

Place.	Latitude.	Longitude.	Altitude.	Years of observation.	Authority : Zeitschrift für Meteorologie.		B ₀ .	B ₁ .	B ₂ .	ε ₁ .	ε ₂ .	Reduced to sea- level and grav- ity of the par- allel of 45°.		
					B.	S.						B ₀ .	ΔB ₁ .	
	°	'	m.				mm.	mm.	mm.	°	'	mm.	mm.	
Port of France.....	— 22	30	166 39	7		4	460	762.8	2.5	0.3	196	188	761.4	
Jerusalem	31	47	37	762	3.5	4	166	695.6	1.9	0.8	21	330	759.0	2.2
Tahiti	— 17	32	— 149 34	0	5	4	528	759.7	1.5	0.3	233	208	758.1	
Adelaide.....	— 34	57	138 38	01	7	5	121	759.5	2.6	0.5	104	215	758.8?	
Hobarttown	— 42	52	147 27	32	20	5	121	756.1	1.6	0.6	134	194	758.9	0.1
Melbourne.....	— 37	49	144 58	37	10	5	99	759.4	2.5	0.3	113	195	762.2	0.2
Calcutta	22	33	88 21	6	16	5	247	756.4	6.5	0.8	6	288	755.9	
Capetadt.....	— 33	56	18 27	0	20	5	428	762.9	2.8	0.2	205	48	762.1	
Höhenpeisenburg	47	48		914	13	5	509	676.9	1.9	0.7	228	52	761.6	3.3
Natal	— 29	36	30 2	638.5	8	5	375	708.3	2.7	0.2	186	352	761.0	0.8
Manritine.....	— 20	10	57 29	9	7	5	552	763.4	4.1	0.4	211	278	762.7	
Brisbane.....	— 27	28	153 6	44	2.5	5	505	762.1	3.0	0.4	193	160	764.5	0.1
Puerto Monti	— 41	30	— 72 52	10	1.5	5	399	761.6	0.6	1.6	274	240	762.2	
Santiago	— 33	26	— 70 37	569	6.5	5	441	717.1	1.4	0.3	211	198	766.1	1.3
Port Said.....	31	18	32 18	3	3	5	228	760.7	3.5	0.7	15	232	760.0	
Ismailia	30	38	32 13	7.6	3	5	228	760.0	3.8	0.8	17	287	759.7	
Suez	29	57	32 32	6	3	5	228	760.9	3.8	0.7	11	291	761.5	
San Francisco	37	48	122 23	0	9	5	642	763.7	2.4	0.1	25	327	763.2	
Sacramento.....	38	35	121 28	25	9	5	642	761.5	3.1	0.3	24	331	762.5	
Palermo	38	7	13 21	70.3	78	5	614	754.6	0.5	0.4	266	9	760.3	0.2
Sereno	— 29	55	71 17	18	2	6	26	760.3	2.0	0.6	215	221	760.9	
New Westminster	49	13	122 53	16.5	2	6	76	762.0	0.5	1.2	95	104	763.8	
Mariotta.....	39	25	81 29	177	40	6	79	743.8	0.9	0.3	275	56	759.2	0.8
Sydney	— 33	52	151 11	47	11	6	84	762.7	2.9	0.6	164	173	765.9	0.1
Mendoza.....	— 32	51	67 32	8	10	6	138	760.6	1.8	0.6	191	141	760.5	
St. Martine.....	45	32	— 73 36	36	10	6	145	756.3	1.6	0.8	289	350	759.6	0.2
Lima	— 12	3	— 79 29	152	1	6	176	747.1	2.2	0.4	249	224	758.4	0.3
Rio Janeiro.....	— 22	54	— 43 20	64	6	6	184	757.4	3.0	1.0	187	16	761.6	0.1
Krasnojarsk	56	0	92 50	(?)	10.	6	223	758.1	6.9	0.8	1	178		
Osaka	34	20	135 10	(?)	1	6	251	761.1	4.1	0.9	5	220		
Scutari	41	0	29 3	18.3	5	6	298	760.3	2.1	0.9	12	220	761.7	
Tercira	38	36	— 27 15	53.8	6	6	310	761.0	1.9	0.9	201	64	765.3	0.1
St. Miguel	37	44	— 26 55	20	6	6	310	764.5	2.1	0.6	206	79	765.8	
Taschkent.....	41	19	69 16	(?)	1	6	329	763.2	3.7	1.7	54	222		
Auckland.....	— 36	50	174 51	0	12	6	370	761.5	1.2	0.8	92	181	760.9	
Taranaka.....	— 39	4	174 5	0	6	6	370	759.9	1.9	0.5	102	185	759.4	
Nelson	— 41	16	173 19	0	6	6	370	759.6	2.1	0.9	142	56	759.3	
Southland	— 46	18	168 10	0	11	6	370	757.1	2.4	0.3	121	84	757.1	
Somerset	— 10	44	142 16	21.3	3	6	378	758.3	1.6	0.3	222	128	758.4	
Fort No. 1, Syr Daria	45	45	64 27	(?)	3	6	384	758.1	6.1	0.2	8	215		
Phare de Donai.....	50	50	146 47	(?)	2	6	384	747.5	1.7	0.5	352	85		
Praia	14	13	— 23 30	35	1.5	6	384	758.8	0.3	0.6	110	299	760.3	
Horta Fayall.....	38	—	28	20	2	6	411	762.0	0.7	2.7	178	58	763.4	
Newchwang	40	57	121 27	0	1	7	7	765.5	7.4	1.2	4	143	765.3	
Ancud	— 41	51	— 74	15	2	7	11	757.1	1.6	0.8	317	114	758.2	
Bangkok	13	43	100 25	(?)	4	7	23	759.5	2.9	0.3	17	55		
Saigon	10	48	106 40	(?)	1	7	23	760.8	1.2	0.5	20	359		
Decima	32	14	129 42	8	2	7	47	761.3	5.7	0.6	10	240	761.4	
Nafa	26	13	128 44	10	2	7	47	760.4	5.1	0.9	21	289	760.0	
Freemantle	— 32	4	115 45	0	3	7	55	762.7	2.0	1.0	180	243	761.8	
Cairo	29	59	31 18	28.5	5	7	67	759.1	3.6	0.6	9	235	760.7	
Alexandria	31	11	29 53	23	3	7	140	760.0	3.6	0.7	7	217	761.1	0.4
Mannheim	49	20	8 27	116	12	7	144	753.5	0.8	1.1	310	42	764.3	
Belize	17	30	— 88 18	(?)	4	7	160	761.7	1.2	0.8	1	31	760.1	
Strasburg.....	48	34	7 45	39	39	7	238	749.5	0.3	0.7	324	43	762.8	0.5
East Falkland	— 51	41	— 57 42	0	9	7	254	750.3	0.8	1.1	202	196	750.8	
Valparaiso.....	— 33	2	— 71 40	0		7	254	763.9	1.6	1.0	218	218	763.1	

TABLE VII—Continued.

Place.	Latitude.	Longitude.	Altitude.	Years of observation.	Authority: Zeitschrift für Meteorologie.		B ₀ .	B ₁ .	B ₂ .	ε ₁ .	ε ₂ .	Reduced to sea- level and grav- ity of the par- allel of 45°.	
					B.	S.						B ₀ .	Δ B ₁ .
	° ′	° ′	m.				mm.	mm.	mm.	°	°	mm.	mm.
Arva Varalgo	49 15	37 15	490	20	7	300	717.5	0.8	0.4	270	24	761.8	2.1
Gratz	47 4	15 28	371	31	7	316	730.8	1.1	0.8	299	25	764.2	1.8
St. Louis	38 37	90 16	154	14	7	326	749.9	1.7	0.7	336	65	763.3	0.8
Yokohama	35 27	139 40	0	2	7	361	760.6	3.2	1.5	4	230	759.5	
Caracas	10 38	66 55	927	3	7	380	684.5	0.5	0.5	128	224	758.9	0.3
Datschitz	49 5	15 26	465	8	7	381	720.5	1.3	0.5	216	350	762.5	1.9
Batavia	6 11	106 50	8	3	7	399	759.0	0.2	0.1	296	123	757.7	
Magador	31 30	9 45	16.6	4	8	8	762.2	1.2	0.5	21	359	762.8	
Nice	43 41	7 6	0	16	8	13	760.8	0.2	0.6	293	20	760.7	
Schaffburg	47 46	13 26	1767	1.5	8	29	614.0	3.8	1.1	216	77	760.3	2.3
Fernanda Po	3 46	8 36	30	4.5	8	47	757.3	0.5	0.2	233	59	757.0	
Lakainalulu	20 52	156 40	199	1	8	70	747.0	1.1	0.5	91	350	762.6	0.1
Honolulu	21 16	157 52	0	1	8	71	764.1	0.6	0.6	196	307	762.6	
Victoria	22 16	114 10	0	8	8	72	760.7	6.1	0.6	15	285	759.3	
Simla	31 6	77 11	2282	3	8	91	589.1	2.0	1.2	351	220		
San Antonio	29 32	98 24	172	1	8	91	755.6	3.5	1.1	38	342	769.7	0.7
Smyrna	38 26	27 10	0	9	8	123	759.1	2.4	1.1	6	262	759.3	
Madrid	40 24	3 42	655	11	8	188	706.9	0.6	0.8	294	16	763.4	2.4
Providence	41 50	71 23	0	28½	8	208	760.7	0.9	0.3	313	185	760.5	
Canton	23 8	113 17	12	10	8	219	762.1	6.1	1.0	13	325	761.8	
Victoria Peak	22 17	114 10	532	2.7	8	219	717.7	3.5	0.5	14	301	761.7	1.1
Mangós	3 8	60 0	37	1	8	269	757.6	1.2	0.3	148	90	758.9	
Havana	23 8	82 28	19.3	14	8	270	761.3	1.1	1.3	32	29	761.8	
Vicenna	48 12	36 22	195	90	8	281	744.9	1.0	0.3	326	60	762.7	0.7
Gorée (Cape Verde)	14 40	17 26	6	4	8	301	757.7	0.6	0.5	49	19	756.5	
Manilla	14 36	128 21	33	5	8	334*	755.0	2.2	0.5	32	333	756.3	
Murcia	37 59	1 7	43	9	9	8	759.7	1.0	1.0	2	14	763.1	
St. George d'Elmira	5 5	1 20	18	3	9	44	759.4	1.6	0.6	220	49	759.1	
Christiansburg	5 36	0 10	20	15	9	45	759.1	1.5	0.8	216	33	758.9	
Arica (Peru)	18 25	70 22	0	1	9	60	761.8	1.3	0.2	222	255	760.2	
Cairo	29 57	31 18	37.8	4	9	61	758.1	3.5	0.9	11	284	760.5	
Gibraltar	36 6	5 21	15	14	9	77	763.4	1.0	0.8	6	8	764.1	
San Fernando	36 28	6 12	29	16	9	77	761.7	1.0	0.8	1	19	763.7	
Santiago	42 53	8 30	273	13	9	111	739.4	0.1	1.2	85	34	763.5	0.6
Oviedo	43 23	5 52	236.5	18	9	111	742.7	0.6	1.0	233	51	763.7	0.6
Leipzig	51 20	12 33	118	40	9	128	751.4	0.5	0.5	257	33	762.6	0.4
Bermuda	32 23	64 40	0	12	9	123	763.9	0.3	1.4	193	32	763.1	
Innsbruck	47 16	11 24	574	40	9	185	707.4	0.6	0.4	256	69	758.3	2.3
Vancouver Island	45°-50°	120°-125°	0	3	9	187	762.4	0.2	0.8	260	313	762.6	
Zaragoza	41 39	1 0	184	9	9	218	743.5	0.9	1.3	5	25	759.6	0.6
Valadolid	41 39	4 47	760	9	9	218	762.2	0.3	1.0	225	39	768.2	2.7
Vera Cruz	19 12	96 9	8	4	9	238	761.8	2.0	0.8	339	40	761.0	
Bodenbach	50 43	14 12	142	46			749.4	0.9	0.7	322	16	762.7	0.6
Biofield (Barbadoes)	13 4	59 37	336.5	21	9	320	736.3	0.4	0.2	108	60	763.2	0.1
Benares	25 20	82 0	80	10			750.4	8.1	0.3	31	237	756.0	0.2
Rurki	29 52	77 56	268	9			734.0	6.8	0.6	2	234	755.7	0.7
Alessandria	44 54	8 37	98	17			752.6	0.7	0.7	352	66	761.5	0.5
Pola	44 52	13 51	31.7	10	10	28	758.7	0.4	0.4	256	354	761.5	0.1
Ancona	43 38	13 31	0	10	10	28	761.8	0.1	0.4	217	20	761.7	
Nagasaki	31 44	129 42	37	1	10	96	762.6	6.3	1.0	20	275	765.1	
Capiapo	27 25	70 23	395	4	10	111†	728.2	1.2	0.3	210	230	761.5	0.7
Buenos Aires	34 37	58 21	31	4	10	118	761.8	3.0	1.1	188	177	763.9	
Alexandria	31 12	29 54	19	2	10	132	756.4	2.7	0.8	18	96	757.2	
Gandokoro	4 55	31 28	465	1	10	192	721.3	1.3	0.9	217	330	758.1	0.5
Chartum	15 36	32 36	388	1	10	192	724.6	1.3	0.3	12	355	755.1	1.0

* Including previous one year, B. v, S. 67.

† Including the previous two years, B. vi, S. 28.

TABLE VII—Continued.

Place.	Latitude.	Longitude.	Altitude.	Years of observation.	Authority: Zeitschrift für Meteoro-logie.		B ₀ .	B ₁ .	B ₂ .	ε ₁ .	ε ₂ .	Reduced to sea-level and gravity of the parallel of 45°.		
					B.	S.						B ₀ .	ΔB ₁ .	
	°	'	m.				mm.	mm.	mm.	°	'	mm.	mm.	
Chnr.....	46	51	9 31	603	15	10	195	700.9	1.4	1.0	243	20	763.5	2.2
Eger.....	50	5		455	11	10	338	720.8	1.1	0.7	214	27	761.6	1.8
St. Louis.....	16	1	— 18 31	5	1	10	381	759.3	2.4	0.7	60	357	758.0	
Port Blair.....	11	41	92 42	18.6	7	11	28	756.7	2.0	0.3	11	122	756.6	
Hanan.....	50	8	8 55	102	6	11	29	754.1	1.0	0.8	262	9	763.7	
Toronto.....	43	39	— 79 23	104	31	11	32	752.3	0.8	0.4	311	117	761.5	
Superior City.....	45	40	— 92 8	197	10	11	58	744.3	0.8	0.5	0	100	762.6	
Marquette.....	46	36	— 87 36	207	10	11	58	743.4	0.4	0.5	288	156	762.7	
Detroit.....	42	21	— 83 7	180	10	11	58	746.1	1.2	0.6	306	103	762.4	
Charlotte.....	43		87	87	10	11	58	754.4	0.9	0.4	328	126	762.4	
San José de Costa Rica..	9	56	— 84 0	1145	8	11	107	668.9	0.1	0.1	90	101	760.5	0.3
Spitzbergen.....	79	53		12	1	11	123	758.2	3.3	2.3	185	262	761.2	
Sabine Island.....	74	32	16 4	0?	1	11	123	758.9	2.2	1.5	74	130	760.6	
St. Martins de Hinx.....	43	47		40?	10	11	125	760.3	0.3	0.7	270	43	763.8	
Mexico.....	19	25		2278	2	11	185	787.0	0.3	0.6	285	67	760.5	2.6
Porto.....	41	8	— 8 37	85	9	11	202	754.7	0.4	1.1	317	16	762.0	1.8
Guerdo.....	40	32	— 7 16	1039	9	11	202	675.3	1.3	1.1	219	20	763.5	3.2
Campo Major.....	39	1	— 7 5	288	9	11	202	737.7	0.8	1.0	345	10	762.6	0.9
Lagro.....	37	7	— 8 25	12	7	11	202	762.5	1.1	0.9	7	12	763.0	
Angra.....	38	36	— 27 15	54	8	11	202	761.0	0.7	1.8	121	70	765.4	0.1
P. Delgado.....	37	40	— 25 55	20	7	11	202	764.5	1.6	0.9	209	38	765.8	
Funchal.....	32	38	— 16 55	25	8	11	202	763.0	0.2	1.0	32	20	764.4	
Hokitika.....	42	42	172 39	0	9.5	11	223	760.3	1.2	1.4	61	160	760.1	
Christ Church.....	43	32	170 59	0	11.5	11	223	759.1	2.0	1.1	115	174	759.0	
Corfu.....	39	38	19 53	0	11	11	283	761.6	1.0	0.3	339	235	761.2	
Nicolajesk.....	63	8	140 43	0	11	11	383	756.7	2.8	1.4	20	150	757.2	
Atlantic Ocean.....	48	0				10	158	760.6	1.4	1.5	196	0	761.0	
	43	0				10	158	762.3	1.6	1.5	191	52	762.4	
	38	0				10	158	763.8	2.0	1.8	176	39	763.6	
	33	0				10	158	765.1	1.4	1.3	167	36	764.6	
	28	0				10	158	765.2	1.0	1.6	125	40	764.4	
	23	0				10	158	764.2	0.1	1.9	130	30	763.0	
	18	0				10	158	762.6	0.2	0.3	79	351	761.3	
	13	0				10	158	761.3	0.7	0.3	87	20	759.7	
	8	0				10	158	760.4	0.4	0.3	183	64	758.7	
	3	0				10	158	760.2	1.1	0.3	218	66	758.5	
	0					10	158	760.3	1.4	0.5	223	71	758.6	
	— 3	0				10	158	760.6	1.4	0.3	225	59	758.9	
	— 8	0				10	158	761.4	1.4	0.3	230	53	759.7	
	— 13	0				10	158	762.5	1.7	0.8	231	358	761.0	
	— 18	0				10	158	763.6	1.9	0.8	221	16	762.2	
	— 23	0				10	158	764.3	1.9	0.8	213	43	763.2	
	— 28	0				10	158	764.3	2.1	0.3	214	236	763.5	
	— 33	0				10	158	763.4	1.4	0.2	236	128	762.9	
	— 38	0				10	158	762.2	0.5	0.2	238	198	762.0	
	— 43	0				10	158	757.5	1.1	0.9	263	172	757.6	
Atlantic Ocean.....	10 to 8 N.	20 to 30 W.	0		11	193	761.2	1.0	0.5	162	2	759.3		
	8 to 6 N.	20 to 30 W.	0		11	193	760.8	0.6	0.1	170	41	758.9		
	6 to 4 N.	20 to 30 W.	0		11	193	760.6	0.8	0.2	212	90	758.6		
	4 to 2 N.	20 to 30 W.	0		11	193	760.9	1.4	0.5	212	50	758.9		
	2 to 0	20 to 30 W.	0		11	193	760.9	1.6	0.7	219	52	758.9		

TABLE VII—Continued.

Place.	Latitude.	Longitude.	Altitude.	Years of observation.	Authority: Zeitschrift für Meteoroelogio.		B_0 .	B_1 .	B_2 .	ϵ_1 .	ϵ_2 .	Reduced to sea-level and gravity of the parallel of 45° .	
					B.	S.						B_0 .	ΔB_1 .
	° /	° /	m.				mm.	mm.	mm.	°	°	mm.	mm.
Around the Cape of Good Hope [*]	— 31 0	0 to 20 E.	0		9	255	764.4	2.2		225		763.4	
	— 33 0	0 to 20 E.	0		9	255	763.3	2.7		247		762.5	
	— 35 0	13 to 35 E.	0		9	255	761.8	2.0		242		761.1	
	— 37 0	13 to 35 E.	0		9	255	761.0	1.4		255		760.5	
	— 39 0	13 to 35 E.	0		9	255	761.4	0.8		249		761.0	
	— 41 0	13 to 35 E.	0		9	255	760.1	1.2		266		759.9	
	— 43 0	13 to 35 E.	0		9	255	758.3	1.3		236		758.2	
	— 45 0	13 to 35 E.	0		9	255	756.4	2.0		232		756.4	
	— 46 30	13 to 35 E.	0		9	255	754.1	1.7		285		754.3	

* Deduced from the average of about 1,000 observations each.

26. The results in the following table are deduced from the monthly means contained in Buchan's paper on the Mean Pressure of the Atmosphere and the Prevailing Winds on the Globe,* for places not contained in the preceding tables.

TABLE VIII.

Place.	Latitude.	Longitude.	Altitude.	Years of observation.	B_0 .	B_1 .	B_2 .	ϵ_1 .	ϵ_2 .	For sea-level and gravity of the parallel of 45° .	
										B_0 .	ΔB_1 .
	° /	° /	m.		mm.	mm.	mm.	°	°	°	mm.
Armagh, Ireland	54 21	— 6 49	64	11	752.7	1.4	0.9	191	312	759.1	0.2
Belfast, Ireland	54 36	— 5 56	0	11	759.0	1.3	1.0	174	315	759.6	
Cork, Ireland	51 53	— 8 28	8	11	759.5	1.4	1.0	190	347	760.7	
Aberdeen, Scotland	57 9	— 2 7	34	11	754.9	1.8	1.4	170	292	758.8	
Glasgow, Scotland	55 3	— 4 18	55	11	752.6	1.5	1.3	179	296	758.3	0.1
Milno-Graden, Scotland	55 0	— 2 12	31	11	755.9	1.6	1.3	173	298	759.4	
Liverpool, England	53 25	— 2 59	0	11	759.2	1.2	1.3	178	326	759.7	
Norwich, England	52 38	1 18	0	11	760.5	0.6	1.5	198	321	761.0	
Helston, England	50 7	— 5 16	32	20	759.0	0.6	0.7	163	49	762.2	
Geneva, Switzerland	46 12	6 9	407	25	726.4	1.1	1.1	273	36		1.4
Turin, Italy	45 4	7 41	279	74	739.1	1.7	0.6	214	56	764.0	1.0
Rome, Italy	41 54	12 28	0	15	761.7	0.2	1.8	301	58	761.5	
Malta, Italy	35 54	14 31	0	6	762.5	0.3	0.2	276	68	761.9	
Bologna, Italy	44 30	11 21	74	40	755.1	0.6	0.6	350	323	761.8	0.3
Krakau, Austria	50 4	19 55	216	19	742.5	1.3	0.6	290	336	762.5	0.8
Kremsmünster, Austria	48 3	14 6	283	19	728.0	1.1	0.9	285	20	762.4	1.2
Szegedin, Austria	46 15	20 6	84	12	754.0	2.0	0.2	324	312	761.8	0.3
Tesina, Austria	43 11	16 25	19	9	759.1	0.5	0.3	300	0	760.6	
Munich, Bavaria	48 9	11 34	511	10	716.0	1.2	1.1	254	2	761.3	1.6
Königsberg, Prussia	54 43	20 29	22	10	758.5	0.1	1.0	225	239	761.2	
Dantzic, Prussia	54 21	18 41	9	32	760.3	0.4	0.2	45	148	761.7	
Corfu, Greece	39 39	19 55	0	6	761.8	1.2	0.4	331	217	761.5	
Barnaul, Russia	53 20	83 57	122	19	749.3	8.1	1.5	15	183	761.2	0.8
Jakutsk, Russia	62 2	129 14	87	13	753.8	7.2	0.3	0	120	763.3	0.9
Bogolovsk, Russia	59 45	60 2	181	26	741.4	3.2	1.0	1	181	759.4	1.2
Ayansk, Russia	56 27	138 26	(?)	2	756.3	2.0	2.0	33	168		
Peterpanshaven, Russia	53 10	158 32	(?)	1	753.8	3.2	2.3	176	56		0.4

* Transactions of the Royal Society of Edinburgh, vol. xxv.

TABLE VIII—Continued.

Place.	Latitude.		Longitude.		Altitude.	Years of observation.	B ₀ .	B ₁ .	B ₂ .	ε ₁ .	ε ₂ .	For sea-level and gravity of the parallel of 45°.		ΔB ₁ .
	°	'	°	'								m.	B ₀ .	
Irkutsk, Russia	52	17	122	11	382	15	724.2	7.7	1.4	14	195	760.1	2.9	
Udskoi, Russia	54	30	134	28	(?)	1	754.1	5.9	0.6	27	308			
Nerohinsk, Russia	51	19	119	36	650	18	705.1	5.0	1.5	3	120	765.6	5.4	
Pekin, China	39	54	116	26	(?)	14	759.2	9.7	1.0	4	204			
Canton, China	23	12	113	17	(?)	10	759.3	7.0	1.1	14	330			
Shanghai, China	30	4	85	33	0	2	761.7	7.2	0.5	6	206	760.7		
Tien-Tsin, China	39	9	117	16	9	1	761.6	7.6	3.5	45	117	762.0		
Hong-Kong, China	22	16	114	10	11	6	760.6	6.1	0.9	15	289	760.2		
Macao, Pelew Islands	22	15	113	36	(?)	1	763.2	5.3	0.3	8	170			
Hakodadi, Japan	41	48	140	47	46	4½	755.4	2.3	2.4	13	194	759.3		
Mooltan, Hindostan	31	11	71	33	137	6	745.7	8.3	0.8	3	213	756.7		
Bombay, Hindostan	18	54	72	48	11	14	757.1	3.5	0.4	5	194	756.5		
Madras, Hindostan	13	4	80	19	8	27	758.4	3.9	0.4	10	69	757.3		
Colombo, Hindostan	6	56	79	50	0	6	758.7	0.9	0.3	352	69	756.8		
Trivandrum, Hindostan	8	31	77	0	40	8½	754.1	1.1	0.4	349	62	755.7		
Upernavik, Greenland	72	48	55	53	5	5	752.8	1.5	0.8	160	234	754.9		
Jacobahaven, Greenland	69	12	51	0	3	9½	755.6	2.2	1.3	159	213	757.4		
Godthaab, Greenland	64	10	51	53	5	5	756.5	2.5	1.8	174	224	758.2		
Baffin's Bay (Arctic)	72	30	(Various)		0	1	755.6	3.7	3.0	146	215	757.3		
Van Rensselaer (Arctic)	78	37	73	0	0	2	756.3	1.9	1.2	90	251	758.1		
Port Foulke (Arctic)	78	18	73	0	2	1	757.5	1.8	3.5	47	267	759.5		
Port Kennedy (Arctic)	72	1	94	0	0	1	760.4	3.9	2.6	72	213	762.0		
Boothia Felix (Arctic)	70	3	95	0	0	2	760.5	1.7	1.7	108	257	762.1		
Mellville Island (Arctic)	75	40	112	3	0	1	758.4	2.7	1.8	75	281	760.2		
Port Bowen (Arctic)	73	13	88	54	0	1	759.1	2.8	3.1	73	235	760.7		
Sitka, Alaska	56	50	135	0	6	17	754.7	3.9	0.6	188	36	756.1		
Esquimaux Harbor	48	25	123	27	0	1	763.3	0.2	0.8	124	83	763.6		
Astoria, Oregon	46	8	123	48	(?)	2½	762.7	0.9	0.2	147	114	762.8		
Saint John's, Newfoundland	47	35	52	43	0	6	759.6	2.5	0.9	217	301	759.8		
Halifax, Nova Scotia	44	39	63	37	0	4	760.0	2.2	1.2	214	323	760.2		
Do	44	39	63	37	0	2	756.3	1.6	0.3	190	239	760.2		
Albion Mines, Nova Scotia	45	34	62	42	0	10	754.7	0.9	0.2	234	148	758.4		
Quebec, Canada	46	48	71	12	0	3½	761.5	1.0	0.6	354	94	761.6		
Kingston, Canada	44	14	76	31	0	4½	761.3	0.7	0.4	353	105	761.2		
Hamilton, Canada	43	15	79	57	99	11	753.5	1.2	0.3	242	9	762.4	0.4	
Gardiner, Me	44	11	69	46	28	5	757.0	1.2	0.5	290	350	759.8	0.1	
Steuben, Me	44	28	67	50	15	6	759.6	1.1	0.7	265	19	761.0		
Amherst, Mass	42	22	72	34	81	6	755.1	1.6	0.3	355	53	762.3	0.3	
New Bedford, Mass	41	39	70	56	28	6	759.5	1.1	0.7	277	51	761.9		
Nantucket, Mass	41	16	70	6	9	6	761.6	1.0	0.6	289	40	762.1		
Burlington, Vt	44	29	73	11	106	5	751.9	1.1	0.6	261	20	761.6	0.6	
Rochester, N. Y	43	8	77	51	158	4	746.7	0.9	0.6	354	51	761.1	1.1	
Harrisburg, Pa	40	16	76	50	85	6	755.5	1.6	0.5	314	62	762.9	0.5	
Washington, D. C	38	36	76	58	22	11	761.7	1.5	0.5	339	51	761.7		
Savannah, Ga	32	5	81	7	13	6	763.3	1.4	0.6	345	38	762.5		
Jacksonville, Fla	30	30	82	0	4	6	764.6	1.6	0.8	359	29	764.0		
Columbus, Miss	33	30	88	29	70	4	757.9	2.0	0.8	347	30	763.5	0.2	
Glenwood, Tenn	36	28	87	13	140	6	751.0	1.5	0.8	339	55	764.1	0.4	
Cincinnati, Ohio	39	6	84	28	155	4	749.0	1.8	0.5	345	57	762.5	0.8	
New Harmony, Ind	38	8	87	50	98	6	753.5	1.6	0.7	346	68	761.8	0.5	
Des Moines, Iowa	42	30	90	52	207	6	744.7	1.3	0.8	325	70	763.1	1.1	
Nassau, West Indies	25	4	77	22	4	6	763.7	0.9	1.3	56	20	762.8		
Up Park Camp, Jamaica	18	0	76	56	0	6	761.7	0.8	0.8	56	50	760.1		
Georgetown, British Guiana	6	50	58	8	3	11	760.4	0.6	0.6	135	76	758.8		
Cayenne, French Guiana	4	56	55	39	2	6	760.0	0.7	0.4	182	90	758.2		
Rio de Janeiro, Brazil	22	57	43	7	69	6	757.9	3.0	0.5	186	42	762.5		
Monte Video, Uruguay	34	54	58	33	8	10	760.4	2.0	0.7	196	170	760.4		
Saint Helena	15	55	5	42	12	5	764.3							

27. The results of the following table have been deduced from the monthly means of barometric pressure given in the reports of the Chief Signal Officer of the United States for the years 1872-76 inclusive. The means as given in the reports are reduced to sea-level, but the results here given are also reduced to the gravity of the parallel of 45°.

TABLE IX.

Place.	Latitude.	Longitude.	Altitude.	B ₀ .	B ₁ .	B ₂ .	ε ₁ .	ε ₂ .
	° ' "	° ' "	<i>Feet.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	°	°
Augusta, Ga.....	33 28	—81 53	172	763.6	1.9	0.8	350	20
Baltimore, Md.....	39 18	76 36	45	763.2	1.8	0.2	346	33
Boston, Mass.....	43 20	71 3	142	761.8	0.9	0.4	308	44
Breckenridge, Minn.....	46 16	96 38	966	762.1	3.9	0.8	5	93
Buffalo, N. Y.....	42 53	78 55	666	761.1	1.2	0.3	351	126
Burlington, Vt.....	44 29	73 11	241	761.5	1.6	0.3	329	—58
Cairo, Ill.....	37 0	89 0	367	763.3	2.6	0.5	346	29
Cape May, N. J.....	39 0	74 58	14	763.0	1.5	0.4	344	24
Charleston, S. C.....	32 45	79 57	61	763.5	1.6	0.8	353	20
Chicago, Ill.....	41 52	87 35	668	761.9	2.0	0.5	354	65
Cincinnati, Ohio.....	39 6	84 30	596	763.1	2.4	0.4	346	38
Cleveland, Ohio.....	41 30	81 36	688	761.8	1.5	0.3	358	82
Davenport, Iowa.....	41 30	90 36	603	762.4	2.8	0.6	352	55
Detroit, Mich.....	42 18	83 0	644	761.5	1.7	0.5	359	94
Duluth, Minn.....	46 48	92 6	643	761.5	2.3	0.4	10	134
Escanaba, Mich.....	46 36	87 6	619	761.3	1.4	0.6	1	104
Fort St. Michael's, Alaska (2 years' observations) ..	63 28	161 45	0	759.8	2.1	1.5	34	70
Galveston, Tex.....	29 19	94 46	39	762.3	1.9	0.8	353	7
Grand Haven, Mich.....	43 5	86 13	616	761.4	1.6	0.3	350	90
Indianapolis, Ind.....	39 42	86 6	747	762.2	2.3	0.3	346	57
Jacksonville, Fla.....	30 15	82 0	23	763.5	1.6	0.9	2	11
Kokomo, Iowa.....	40 18	91 30	584	761.4	2.8	0.4	353	68
Key West, Fla.....	24 36	81 48	32	762.3	1.1	1.0	35	25
Knoxville, Tenn.....	35 56	83 58	993	763.2	2.3	0.6	347	12
Lake City, Fla. (3 years' observations) ..	30 6	82 42		762.9	1.4	0.9	352	0
Leavenworth, Kans.....	39 21	94 44	813	761.5	3.6	0.5	350	49
Louisville, Ky.....	38 0	85 25	426	762.5	2.4	0.4	348	46
Lynchburg, Va.....	37 18	85 54	651	763.3	2.1	0.5	344	12
Marquette, Mich.....	46 33	87 23	666	761.3	1.5	0.7	27	95
Memphis, Tenn.....	35 8	88 0	299	763.5	2.6	0.5	347	20
Milwaukee, Wis.....	43 3	87 57	672	762.0	1.6	0.4	346	84
Mobile, Ala.....	30 42	87 59	39	763.6	1.9	0.8	0	5
Nashville, Tenn.....	36 10	86 49	504	763.4	2.6	0.5	346	24
New London, Conn.....	41 22	72 9	38	762.5	1.0	0.3	323	63
New Orleans, La.....	29 57	90 0	56	762.9	1.8	0.7	358	5
New York, N. Y.....	40 42	74 1	166	762.7	1.3	0.4	339	49
Norfolk, Va.....	36 51	76 19	56	763.2	1.4	0.4	330	308
Omaha, Nebr.....	41 26	96 0	1055	760.6	3.6	0.5	356	62
Oswego, N. Y.....	43 28	76 35	299	761.7	0.9	0.5	343	94
Philadelphia, Pa.....	39 57	75 12	47	763.2	1.6	0.3	344	35
Pittsburgh, Pa.....	40 32	80 2	791	762.0	2.2	0.7	357	63
Portland, Me.....	43 40	70 14	584	761.2	0.4	0.4	297	56
Portland, Oreg.....	45 30	122 27	90	764.4	0.4	0.6	45	260
Punta Rassa, Fla.....	27 0	82 18	17	763.2	1.1	0.8	24	24
Rochester, N. Y.....	43 8	77 51	584	761.2	1.2	0.4	340	320
San Diego, Cal.....	32 44	117 6	62	761.8	1.8	0.4	37	298
San Francisco, Cal.....	37 48	122 26	60	762.5	1.6	0.2	32	290
Savannah, Ga.....	32 5	81 8	71	763.6	1.8	0.8	356	9
Shreveport, La.....	32 30	93 45	229	762.5	2.2	0.5	359	11
Saint Louis, Mo.....	38 37	90 16	557	762.5	2.5	0.5	348	55
Saint Paul, Minn.....	44 53	93 5	794	761.0	2.7	1.0	358	116
St. Paul's Island, Alaska (4 years' observations) ..	57 3	170	0	755.9	3.0	1.6	159	73
Toledo, Ohio.....	40 39	83 32	531	761.7	1.9	0.3	346	36
Vicksburg, Miss.....	32 24	91 0	280	764.1	2.4	0.6	350	351
Washington, D. C.....	38 53	77 1	106	763.0	2.0	0.3	355	340
Wilmington, N. C.....	34 11	—78 10	74	763.5	1.6	0.7	353	0

28. With the values of B_0 in the preceding tables, Charts I and II have been constructed, showing the mean barometric pressure, reduced to the gravity of the parallel of 45° , for the northern and southern hemispheres, by giving the positions of the isobars for each 2^{mm} . These isobars represent the mean pressures very accurately in Europe and the United States of America, the greater part of the North Atlantic, and many other places where the observations suffice to lay them down accurately; but throughout all the interior of Asia, Africa, and South America, and the greater part of the great oceans, where there are but few observations, and these not reliable in many cases on account of the uncertainty of altitude above sea-level, and the lack of comparisons of the barometers used with any standard of comparison, of course the true positions of these lines are uncertain, but nowhere entirely conjectural, since we can derive much aid from theory and analogy in laying down these lines for those parts of the earth's surface for which we have few or no observations. As reliable observations multiply, and are obtained for those parts of the earth for which we have yet no observations, of course the positions of the lines as laid down in these charts will be found to be somewhat in error for all places for which we have not yet sufficient observations to determine them; but it is thought that the errors in general will be found to be small.

The arrows on these and the following charts denote the prevailing directions of the wind. These are given, not from observation, but from theoretical considerations of the relations between the gradients and the directions and velocities of the winds, to be explained in a subsequent part of this work. The winds, as represented on these charts, are the resultants of all the winds for the whole year, which can now be laid down more accurately from a knowledge of the isobaric lines than from observations, which in most parts of the earth consist merely in the observation of the relative frequency of the winds from the different points of the compass. The prevailing direction of the wind, as obtained from such observations, may be very different from the resultant obtained by Lambert's formula from observations of the true velocities and directions of the wind through the year.

29. With the values of B_1 in the preceding tables, reduced to sea-level by means of the values of ΔB_1 where the monthly means of the observations were not given for sea-level, Charts III and IV have been constructed, representing the coefficients of the annual inequality of barometric pressure over the whole globe. These coefficients are accurately represented by the charts for all portions of the earth where the observations were sufficient for their determination; but, of course, there is the same uncertainty with regard to them where few or no observations have been made which there is with regard to the mean pressures.

Where the signs of these coefficients as given on the charts are positive, the maximum of the barometric pressure occurs in the winter and the minimum in the summer. It is seen from the chart of the northern hemisphere that these signs are mostly positive on land and negative on the ocean, especially on the middle parallels of latitude. This arises from the higher temperature of the air in summer and lower temperature in winter on land than on the ocean. The line of no annual inequality of barometric pressure passes over Norway and Sweden and a little east of London, touching upon France and Portugal, having its most southern point in the middle of the Atlantic, a little south of the parallel of 20° , and then, curving northward, passes over the eastern part of New England in America.

On account of the great extent of continent and the great extremes of temperature in the interior of Asia, this coefficient of the annual inequality amounts to about 10^{mm} , or a range of 20^{mm} between winter and summer, while in America it amounts at the maximum to only about one-third as much. This difference between Asia and North America does not depend so much upon the difference in the extremes of temperature of the two countries, which is inconsiderable, as upon the difference in the extent of the two continents.

The lines on Charts III and IV represent the gradients which in winter have to be added to the gradients given on Charts I and II to obtain the gradients for that season, and these in summer are completely reversed, and hence the steeper these gradients the greater are the monsoon influences in the different parts of the globe. These, it is seen, are very great in the southern part of Asia.

In the southern hemisphere, on account of the small extent of land and the small range of temperature between winter and summer, the coefficient of the annual inequality of barometric pressure is small and negative so far as we have observations to determine it, and nearly the same

in all longitudes on the same parallel of latitude. Its being negative shows that the maximum occurs in winter, that is, during the summer of the northern hemisphere. About the parallel of 55° , this coefficient seems to vanish, beyond which it probably becomes positive, making the maximum of pressure in the summer as in the northern parts of the Atlantic and Pacific Oceans of the northern hemisphere.

30. The values of ϵ_1 in the preceding tables are very various, depending upon the want of sufficient observations in many cases to eliminate the abnormal inequalities and bring out the true value, especially in such cases as give a small value of the coefficient B_1 , which is often less than the possible error of observation, when the value of ϵ_1 is of course indeterminate, and may have any value whatever. Taking the average of all the values of ϵ_1 in Table VI belonging to coefficients greater than 3.0^{mm} , we get $\epsilon_1 = 9^\circ$, which makes the maximum of barometric pressure in Europe occur about the 9th of January, a little earlier than the minimum of temperature. This is perhaps the most probable value of ϵ_1 for all stations in Europe, the variations from this value of ϵ_1 in the different stations being merely possible errors of observation, though they may possibly depend in some measure upon local causes. To this value, the values of ϵ_1 for other places in Europe in the other tables seem also to point in cases in which the coefficient is sufficiently large for the values of ϵ_1 to be determined approximately from observation.

Where stations have great altitude above sea-level, as in the case of Höhenpeisenburg in Table VII, the value of ϵ_1 is such as to make the maximum of the barometric pressure occur in summer instead of winter, in which case we can change ϵ_1 by 180° , and consider the value of B_1 as negative. Applying the correction then in the column headed by ΔB_1 to reduce this coefficient to sea-level, it becomes positive, and makes the maximum of pressure fall in the winter. In the case of Höhenpeisenburg, we get $-1.9^{\text{mm}} + 3.3^{\text{mm}} = 1.4^{\text{mm}}$ for the coefficient of annual inequality, with the value of $\epsilon_1 = 228^\circ - 180^\circ = 48^\circ$, making the maximum occur after the middle of February. In applying the reduction to sea-level, ΔB_1 , it is supposed for convenience that the maximum of pressure coincides with the minimum of temperature, which, we have seen, is not strictly correct, but the errors in these small reductions arising from this cause are generally very small.

In all cases in which the values of ϵ_1 are such as to throw the maximum of barometric pressure into the time of summer of the northern hemisphere, as in the northern parts of the Atlantic and Pacific Oceans and in the southern hemisphere generally, the values of B_1 as entered in the Charts III and IV are considered negative, and in all such cases the values of ϵ_1 must be diminished by 180° , or the negative sign of the coefficient on the chart changed to the positive sign if used with the average value of ϵ_1 given by the tables in such cases, which does not generally differ much from 200° , throwing the maximum of barometric pressure in July.

If we take the average of all the values of ϵ_1 in Table IX, deduced from the observations of the Signal Service of the United States, giving them weights in proportion to the magnitudes of B_1 , and excluding the stations on the Pacific coast, we get $\epsilon_1 = 353^\circ$ for the stations north of the parallel of 40° and $\epsilon_1 = 351^\circ.1$ for the stations south of that parallel; and hence we may put for the United States, except the Pacific coast, $\epsilon_1 = 352^\circ$. This makes the maximum of barometric pressure occur about the 23d of December, and about sixteen days earlier than in Europe, and in both places considerably earlier than the time of the minimum of temperature. This is most probably caused by the greater amount of aqueous vapor in the atmosphere in the spring than in the fall, which causes the maximum of barometric pressure to be earlier.

31. In addition to the *annual* inequality of barometric pressure, there is also a very small semi-annual inequality, as may be seen from an inspection of the columns in the preceding tables headed by B_2 and ϵ_2 . The values of B_2 as given are mostly of the order of the probable, or, at least, possible, errors of the results, as may be seen from the scattering values of ϵ_2 , and hence do not indicate real terms; but if we examine all the larger values of B_2 , we find that the corresponding values of ϵ_2 are such as to indicate real terms, and give a maximum of inequality in the middle of winter and summer, and a minimum in the spring and fall. The coefficient of this inequality seems to be generally less than 1.0^{mm} , but in some places, especially toward the north pole, it appears to be much greater.

The average of these coefficients in Table IX, from the observations of the Signal Service of the United States, excluding the stations on the Pacific coast, give for the stations south of the parallel of 35° , $B_2 = 1.02^{\text{mm}}$, and the corresponding average of the values of ϵ_2 , giving each weight in proportion to the magnitude of the coefficient, is 8° , indicating maxima about the 4th of January and July. In the same manner, we get for the stations between the parallels of 35°

and 40° , $B_2 = 0.53^{\text{mm}}$, and $\varepsilon_2 = 19^\circ.6$. For those stations north of the parallel of 40° , we get $B_2 = 0.48^{\text{mm}}$ and $\varepsilon_2 = 45^\circ.4$. The coefficient, therefore, seems to diminish in the United States with the increase of latitude, and the times of maxima to become later, being, in the northern part of the United States, about the 23d of January and July.

32. If we take from Charts I and II the mean barometric pressures for each fifth parallel of latitude and each tenth degree of longitude, and take the averages with regard to the different longitudes, we get the results contained in the first column headed B_0 in Table X, which are the mean pressures for the corresponding latitudes in the first column. It is seen that there is a minimum pressure about 6° north of the equator, a maximum in the northern hemisphere near the parallel of 35° , and in the southern hemisphere near the parallel of 28° . There is also a minimum at the parallel of 65° in the northern hemisphere, arising from the two great depressions of barometric pressure in the northern parts of the Atlantic and Pacific Oceans.

From the differences of B_0 , with the irregularities a little smoothed off, the gradients in the column headed G are obtained, which express, in millimeters, the differences of barometric pressure, corresponding to a distance of one degree of a circle having the mean radius of the earth, or 111111 meters.

By taking the averages of all the values of B_1 , taken from Charts III and IV, for each tenth degree of longitude, we get the results contained in the fourth column of Table X, from the differences of which, smoothed off a little, we get the corresponding gradients in the column headed by g . These latter results, added to those of the mean pressures, give the pressures and gradients for January, and subtracted give those for July. From what we have seen in § 30, the maxima and minima for the average of all longitudes occur about the first of these months, and before the maxima and minima of temperature.

TABLE X.

Lat.	Annual mean.		Annual inequality.		January.		July.	
	B ₀ .	G.	B ₁ .	g.	B ₀ .	G.	B ₀ .	G.
°	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>
80	760.5	-----	-0.06	-----	760.4	-----	760.6	-----
75	760.0	-0.19	+0.19	+0.04	760.2	-0.15	758.8	-0.23
70	758.6	-0.14	0.36	0.05	759.0	-0.09	758.2	0.19
65	758.2	+0.01	0.63	0.06	758.8	+0.07	757.6	-0.05
60	758.7	0.15	0.97	0.06	759.7	0.21	757.7	+0.09
55	759.7	0.20	1.26	0.05	761.0	0.25	758.4	0.15
50	760.7	0.18	1.41	0.03	762.1	0.21	759.3	0.15
45	761.5	0.15	1.53	0.02	763.0	0.17	760.0	0.13
40	762.0	+0.07	1.61	+0.01	763.6	+0.08	760.4	+0.06
35	762.4	-0.03	1.66	0.00	764.1	-0.03	760.7	-0.03
30	761.7	0.18	1.66	-0.01	763.4	0.19	760.0	0.17
25	760.4	0.25	1.61	0.03	762.0	0.28	758.8	0.22
20	759.2	0.21	1.41	0.06	760.6	0.27	757.8	0.15
15	758.3	0.13	1.05	0.09	759.3	0.22	757.3	-0.04
10	757.9	-0.03	+0.50	0.11	758.4	0.14	757.4	+0.08
5	758.0	+0.01	-0.05	0.12	758.0	0.11	757.9	0.13
0	758.0	0.04	0.63	0.12	757.4	-0.08	758.6	0.16
-5	758.3	0.11	1.18	0.11	757.1	0.00	759.5	0.22
10	759.1	0.20	1.70	0.08	757.4	+0.12	760.8	0.28
15	760.2	0.26	2.00	0.06	758.2	0.20	762.2	0.32
20	761.7	0.29	2.22	-0.03	759.5	0.26	763.9	0.32
25	763.2	+0.18	2.36	0.00	760.8	+0.18	765.6	+0.18
30	763.5	-0.08	2.22	+0.03	761.3	-0.05	765.7	-0.11
35	762.4	0.30	1.85	0.06	760.6	0.25	764.2	0.35
40	760.5	0.51	1.41	0.07	759.1	0.44	761.9	0.58
45	757.3	0.73	1.00	0.09	756.3	0.64	758.3	0.82
50	753.2	0.91	-0.50	0.10	752.7	0.81	753.7	1.01
55	748.2	0.97	0.00	+0.10	748.2	-0.87	748.2	-1.07
60	743.4	0.83						
65	739.7	-0.56						
-70	738.0							

With the values of G in this table, the values of D, P contained in equations (15), and in expressions deduced from them in the following chapter, are readily obtained.

33. Chart V shows, by isobaric lines, the mean pressure of the atmosphere in the northern hemisphere for January, in millimeters, reduced to the gravity of the parallel of 45° , and, by arrows, the prevailing directions, or rather the directions of all the resultants for the month. These latter are inserted, as in the case of Charts I and II, from theoretical considerations of the relations between the winds and the isobars, and not from results deduced from actual observations.

Chart VI shows the same for the month of July. The epochs of maxima and minima on the earth's surface generally, from what has been stated, are nearly the 1st of January and July respectively, and not at the times of the least and greatest temperatures.

On the first of these charts, there are two areas of great barometrical depressions over the northern parts of the great oceans, and two areas of high barometer over each of the continents, and consequently having the isobars mostly crowded closely together, with corresponding strong prevailing winds. On the last of these charts, for July, in consequence of the reversal of the annual inequality, it is seen that these areas of low and high barometers are very much smoothed off, and consequently the isobars are much separated, with corresponding small velocities of the prevailing or resultant winds; for the winds in the same latitudes are very nearly inversely as the distances between the isobars, as will be explained in the second part of this work.

As the barometric pressures given on the chart are the observed pressures reduced to those of the gravity of the parallel of 45° , in comparing observed pressures in any part of the world with those given by the charts, the same reduction must first be made. The part of gravity depending upon latitude is expressed by the last term in the expression of g in (13). The effect upon the height of the mercurial column is inversely as that upon gravity, and hence the observed column is too low toward the poles and too high toward the equator. The correction, therefore, to be applied to the observed height for the pressure of 0.76^m is $0.76^m \times 0.00284 \cos 2\theta$, which can be used for all parts of the earth's surface without material error. This reduction is given for each fifth degree of latitude in the following—

TABLE

Latitude.	Reduction.	Latitude.	Reduction.	Latitude.	Reduction.
°	<i>mm.</i>	°	<i>mm.</i>	°	<i>mm.</i>
0	— 2.16	30	— 1.08	60	+ 1.08
5	2.12	35	0.74	65	1.39
10	2.03	40	— 0.37	70	1.65
15	1.87	45	0.00	75	1.87
20	1.65	50	+ 0.37	80	2.03
25	— 1.39	55	+ 0.74	85	+ 2.12

CHAPTER III.

THE GENERAL MOTIONS OF THE ATMOSPHERE.

34. Under the head of "The general motions of the atmosphere" are included all those motions which extend as a system over the whole globe, and depend upon differences of temperature between the equatorial and the polar regions at all seasons, and hence they comprise not only the mean motions of the atmosphere, but likewise the changes in these motions depending upon the seasons; but they do not include those motions or disturbances depending upon permanent differences, for the time, of temperature in different longitudes, upon local disturbances of temperature or of density from any cause, or upon the irregularities of the earth's surface. The conditions to be satisfied in this case are those of equations (15), in which, since differences of temperature in longitude are not considered, $D_t \log a'$ vanishes, and consequently the last term of the second equation. The complete solution of these equations is impossible, both on account of their complexity and the uncertain element of friction entering into them, the laws and the amount of which are unknown. Many important results, however, may be deduced from their consideration and solution in special cases, from which approximate results may be obtained by neglecting the effects of friction, and the latter, with the aid of observation, may be shown in most cases to be very small.

If the temperature and amount of aqueous vapor upon which a depends were the same over all parts of the earth's surface, $D_u a'$ and $D_v a'$ in (15) would vanish, and it is readily seen that the conditions of (15) in this case are satisfied with $D_t u = 0$ and $D_t v = 0$, and consequently with a state of rest and of uniform pressure over the whole globe. And if the atmosphere were set in motion by any external impulse, this motion, in the case of friction, would be speedily destroyed, and a state of rest ensue. *There can be no winds, then, without a disturbance of the static equilibrium by means of a difference of temperature or of aqueous vapor in different parts of the atmosphere.*

35. In the case of no friction, where u is independent of longitude, it is evident that P' is likewise independent of longitude, and the first member of the second of (15) must vanish, as well as the last two terms of the second member, and the equation is reduced to—

$$0 = D_t^2 v + 2 \cos \theta (n + D_t \omega) D_t u$$

Since we have $u = r D_t \theta$ and $v = r \sin \delta D_t \varphi$, this equation may be expressed in the following form:—

$$2 \sin \theta \cos \theta (n + D_t \varphi) D_t \theta + \sin^2 D_t^2 \varphi = 0$$

The integral of this equation is—

$$\sin^2 \theta (n + D_t \varphi) = c$$

in which c is a constant depending upon the initial east or west velocity, or value of $D_t \varphi$, of the particle supposed to be not influenced in its motions by contiguous parts, as implied by putting $F_v = 0$ in the original equation in (15).

If we put θ' and v for the initial values of θ and $D_t \varphi$, we have—

$$c = \sin^2 \theta' (n + v)$$

We shall therefore have, if we suppose the particles to have such an action upon each other as to reduce, in time, the motions of all the particles of the atmosphere upon the same parallel of latitude to the same, and that there is no resistance between the earth's surface and the atmosphere,

$$(37) \quad \dots \dots \dots \int_m \sin^2 \theta (n + D_t \varphi) = \int_m c = \frac{2}{3} (n + v') m$$

in which m is the mass of the atmosphere, and—

$$v' = \frac{1}{m} \int_m \sin^2 \theta v$$

If the initial state of the atmosphere is that of rest relative to the earth's surface, we have v , and consequently $v' = 0$.

The first member of (37) expresses in terms of the earth's radius the sum of all the areas described in a unit of time by a line drawn from the earth's axis to each particle of the atmosphere, and this sum must always remain the same since no mutual actions of the particles upon each other can change it, and the velocity of each particle at the same distance from the earth's axis, that is, upon the same latitude, must always be the same after they have been brought to this state by their mutual actions upon each other. We shall then have—

$$(38) \quad \dots \dots (n + D_t \varphi) = \frac{2n(n + v')}{3 \sin^2 \theta}$$

The first member of this equation represents the angular velocity of a particle of atmosphere around the earth's axis depending upon the velocity of the earth's rotation n and the angular velocity $D_t \varphi$ relative to the earth's surface, and this velocity, it is seen, as the particle moves toward or from the pole, must be inversely as $\sin^2 \theta$, and consequently inversely as the square of the distance from the axis of rotation, and this is independent of any law governing the motion toward or from the pole, just as in the case of the planets, or the motions of any free body controlled by a central force, whatever may be the law of that force. Multiplying both members of (38) by $r \sin \theta$, it becomes the expression of the linear velocity.

From (38) we get, in the case of a state of initial rest relative to the earth's surface, in which case $v' = 0$,—

$$(39) \quad \dots \dots \begin{cases} D_t \varphi = \left(\frac{2}{3 \sin^2 \theta} - 1 \right) n \\ D_t v = r \sin \theta D_t \varphi = r n \left(\frac{2}{3 \sin \theta} - \sin \theta \right) \end{cases}$$

The first of these expresses the angular and the second the linear velocity of eastward motion relative to the earth's surface.

If we put $D_t \varphi = 0$, the first of (39) gives—

$$(40) \quad \dots \dots \sin \theta = \frac{2}{3}$$

from which we get $\theta = 54^\circ 44'$, corresponding to the parallel of $35^\circ 16'$, where there is no east or west motion of the air. All velocities between this parallel and the pole are positive, and those between this parallel and the equator negative.

If we substitute the preceding value of $D_t \varphi$ in the first of (15), and neglect the effect upon P' of a difference of density, and of the inertia of the fluid represented by $D_t^2 u$, we get, since $r D_u \log P'$ is equal to $D_\theta \log P'$, and F_v vanishes in the case of no friction,—

$$\frac{1}{a'} D_\theta \log P' = r^2 n^2 \sin \theta \cos \theta \left(\frac{4}{9 \sin^4 \theta} - 1 \right)$$

By integration, regarding a' as constant, we get—

$$\frac{1}{a'} \log P' = -r^2 n^2 \left(\frac{2}{9 \sin^2 \theta} + \frac{1}{2} \sin^2 \theta \right) + C$$

in which, if we put P'' for the value of P' at the equator, we have—

$$C = \frac{1}{a'} \log P'' + \frac{13}{18} r^2 n^2$$

Hence,

$$(41) \quad \dots \dots \log P' = \log P'' + M a' r^2 n^2 \left(\frac{13}{18} - \frac{2}{9 \sin^2 \theta} - \frac{1}{2} \sin^2 \theta \right)$$

in which the factor M , the modulus of common logarithms, must be used when these are used.

In inelastic fluids we have merely to substitute $P' = g h$ for $\log P'$ in all the preceding equations. Hence, in this case, we get from (41), by putting the first member equal 0, a condition for determining the latitude where the pressure, and consequently h , the height of the fluid, vanishes, and hence a condition for determining the polar limit of the fluid. This limit depends upon the value of P'' , the pressure or the height of the fluid at the equator, and the greater this is the higher the latitude, where h , the height of the fluid, becomes 0.

With the value of $r n$ (§ 14), and the value of $a' = a_0$ given by (20), which is the value belonging to the temperature of 0° , we get $M a' r n^2 = 1.1946$. With this value, and the special value of $\sin^2 \theta = \frac{2}{3}$ in (40), belonging to the latitude where the pressure is a maximum, we get—

$$\log P' = \log P'' + 0.06636$$

If we suppose P'' , the pressure or value of P' at the equator, equal to 0.76^m , we get for the maximum pressure near the parallel of 35° —

$$\log P' = 9.88081 + 0.06636 = 9.94717$$

Hence, at this parallel, $P' = 0.88545^m$, or 0.12545^m greater than at the equator. There is, therefore, in this case a great depression or diminution of the pressure at the equator, an accumulation having its maximum near the parallel of 35° , and almost a vacuum near the poles.

With the value of $r n$ in § 14, the second of (39) gives at the equator, where $\sin \theta = 1$,—

$$D_t v = -154.76^m$$

for the velocity per second of the atmosphere at the equator in this case, or a westward velocity of about 557 kilometers per hour. Toward the poles, where $\sin \theta$ in the denominator of one of the terms in the expression of $D_t v$ (39) becomes small, the eastward velocity becomes very great.

36. We know from observation that in the case of nature in which there is friction, the value of $D_t \phi$, the angular velocity of the atmosphere relative to the earth's surface, is very small in comparison with n , the angular velocity of rotation, and hence may be neglected in comparison with it. In the case, therefore, under consideration now, in which the first member of the second of (15) vanishes, and also the last term of the second member, we have very nearly—

$$(42) \quad \dots \dots \dots 2 n \cos \theta D_t u = - (F_v + D_t^2 v)$$

The first member expresses the deflecting force depending upon the earth's rotation on its axis, and upon $D_t u$, the velocity along the meridian, and it is this force alone which overcomes the friction and inertia of the air represented by the two terms in the second member. At the equator, where $\cos \theta = 0$, we have—

$$D_t^2 v = -F_v \quad \text{or} \quad D_t v = -F_v t$$

As F_v is negative, it tends to destroy all motion which the air might have at the equator, and hence there can be no east and west motion there in the case we are now considering, in which only differences of density between the equatorial and polar regions are considered. Since $D_t u$ in (42) becomes very small toward, and vanishes at, the poles, we must for the same reason have a calm about the poles.

37. If the motions of the atmosphere were not resisted by the earth's surface, the results of the preceding section for the case of no friction could be at once applied to them without any modifications; hence toward the poles there would be a very rapid motion of the atmosphere eastward and in the equatorial regions toward the west, and the atmosphere would be very much depressed at the equator, and almost vanish from the polar regions, and become very protuberant about the parallels of 35° . Although these results, when applied to the atmosphere, must be very much modified on account of the resistances of the earth's surface, yet they will be of great advantage in explaining its general motions; for as there can be no resistance until there is motion, the atmosphere must have a tendency to assume, in some measure, the same motions and figure of outline as in the case of no resistances. Hence toward the poles, the general motions of the atmosphere at the earth's surface must be toward the east, and in the torrid zone toward the west; but as these motions, in consequence of the resistances, are small in comparison with those in the case of no resistances, instead of the atmosphere having a great depression at the equator, and almost entirely receding from the poles, there are only comparatively small depressions, as represented in Chart VII, in which the outline of the part representing the atmosphere must be regarded merely as a stratum of equal density in the upper regions. That there are really such depressions at the equator and the poles is shown by the barometric pressures given in Table X, for the pressures are less at the equator and toward the poles, especially the south pole, than in middle latitudes, although the density of the atmosphere at the poles is much greater on account of the lowness of the temperature, and hence there must be considerable polar depressions in the strata of equal density.

Chart VII is intended to represent the mean annual vertical or horizontal surface motions of

the atmosphere in the case of a homogeneous surface over the whole globe, of either all water or all land of the same unevenness, and with the same temperatures at corresponding parallels of the two hemispheres. Hence the equatorial calm-belt coincides with the equator, and the other two calm-belts are found on corresponding parallels, and the whole system of the winds in the two hemispheres is exactly similar. On account, however, of the unequal distribution of land and water in the two hemispheres, these calm-belts are somewhat displaced, being all moved a little toward the north pole; and on account of the unequal distribution of temperature in the different longitudes corresponding to the same parallels of latitude in the same hemisphere, arising from the unequal distribution of land and water, these calm-belts and the general system of the winds are very much deranged, especially in the northern hemisphere, as represented in Charts I and II.

38. As the motion of the atmosphere is east toward the poles and west near the equator, somewhere between the equator and the poles there must be a parallel of latitude where there is no east or west motion. This, in the case of no resistance, and upon the hypothesis of an initial state of rest, we have found to be nearly the parallel of 35° ; but, in the case of nature, in which there are frictional resistances at the earth's surface to the motions of the atmosphere, this parallel depends upon the law of the resistances and the velocities of motion, and hence it cannot be accurately determined from theory. It is evident, however, that the east and west motions of the atmosphere at the earth's surface must be such that the sum of the resistances of each part of the earth's surface multiplied into its distance from the axis of motion must equal 0, else the velocity of the earth's rotation would be continually either accelerated or retarded, which cannot arise from any mutual action between the earth's surface and the surrounding atmosphere. Now, as the part of the earth's surface where the motion of the atmosphere is west is much farther from the axis than the part where it is east, it is reasonable to suppose that the parallel of no east and west motion must be nearer to the equator in this case than the parallel of 35° , unless the eastward velocities toward the poles were very much greater than the westward velocities toward the equator. This is known to be the position of this parallel from observation. In speaking of the east and west motions of the atmosphere, of course only one component of the real motions is understood.

39. Since there is an eastward motion of the atmosphere in the polar regions and a westward motion nearer the equator, the protuberance of the outline of the atmosphere and the increase of pressure in the middle latitudes, with a maximum near the parallel of 30° , is readily explained by the principle given in § 13; for, according to this principle, the eastward motions in both hemispheres give rise to a deflecting force, arising from the earth's rotation toward the equator, and the westward motions nearer the equator to a deflecting force toward the poles, and hence there must be an accumulation of atmosphere having its maximum between these east and west motions.

40. The increase of pressure arising from the accumulation of atmosphere near the parallels of 30° gives the atmosphere at the earth's surface a tendency to flow from beneath this accumulation both toward the equator and the poles, since the motions, and consequently the deflecting forces arising from these motions, and causing this accumulation, are much less near the surface, where friction is greatest, than in the higher strata. But on account of the greater density of the atmosphere toward the poles, it has a tendency also to flow, at the earth's surface, from the poles toward the equator. Between the parallel of greatest pressure and the equator these tendencies combine and produce a strong surface-current which, combining with the westward motion there, gives rise to the well-known northeast wind in the northern and southeast wind in the southern hemisphere, represented on Chart VII, called the trade-winds. But between the parallels of greatest pressure and the poles, these tendencies are opposed to each other, and the one arising from the accumulation of atmosphere being the greater in the middle and polar latitudes and near the earth's surface causes the atmosphere there to flow toward the poles; and this motion, combining with the general eastward motion of the atmosphere in these latitudes, gives rise to the southwest winds in the northern and the northwest winds in the southern hemisphere, which prevail in these latitudes, as represented on Chart VII.

41. Since the atmosphere at the parallel of greatest pressure has no barometric gradient, it can have no north or south motion there at the surface, and consequently D_u in (33) vanishes, and there is no force arising from the earth's rotation to overcome the friction at the surface; and

hence there can be no east and west motion there, and we have what are called the tropical calm-belts. These calm-belts, therefore, must coincide with the belts of maximum pressure, which, for the average of all longitudes, it is seen from Table X, is near the parallel of 35° , being a little farther from the equator in the northern hemisphere than the southern, on account of the unequal distribution of land and water; the land with high mountain-ranges diminishing the east and west motions in the northern hemisphere, upon which the positions of these calm-belts depend.

From what precedes, the mean motions of the atmosphere, unobstructed by inequalities of surface, such as continents and mountain-ranges, would be at the earth's surface nearly as represented in Chart VII, and the calm-belts have positions very nearly as there represented.

42. From the first of (15), we get, by neglecting $D_t \varphi$ in connection with n ,—

$$D_t v = \frac{\frac{1}{a'} D_u \log P' - g h \frac{D_u a'}{a'} + D_t^2 u + F_u}{2 n \cos \theta}$$

With regard to the value of $D_t^2 u$ arising from the inertia of the atmosphere, we know that in all the general motions of the atmosphere it is very small. The interchanging motions of the atmosphere between the equatorial and the polar regions arising from the disturbance of static equilibrium on account of a difference of temperature consist, except near the earth's surface, of a motion in the upper regions toward the poles, and in the lower regions of a motion from the poles, the whole circuit of this motion being performed in a long period of time. Hence the rate of increase or decrease of velocity in any part of the circuit, upon which the value of $D_t u$ depends, is very small.

The greatest velocity of these motions is perhaps not more than 10 kilometers per hour, or 2.778^m per second, and if we suppose all this velocity to have been generated in 24 hours, or 86400 seconds, we shall have, by putting G for the force required to overcome the inertia,—

$$G = D_t^2 u \quad \text{and} \quad D_t u = G t$$

With the value of $D_t u = 2.778^m$, above, and $t = 86400$ seconds, we get—

$$G = \frac{2.778^m}{86400}$$

Now we have g , the accelerating force of gravity, equal to 9811^{mm}, and hence we have—

$$\frac{G}{g} = 0.00000032775$$

This is the ratio between the horizontal force required to overcome the inertia of the atmosphere and the vertical force of gravity, and corresponds to a gradient in the atmosphere of 0.364^m in the distance of one degree, or 11111^m. This multiplied into the ratio between the density of air and mercury, $\frac{1}{10511}$, gives a barometric gradient of nearly 0.0035^{mm}, a quantity which might generally be neglected in comparison with the gradients upon which velocities generally depend.

With the preceding value of G , if we put $\Delta D_t v$ for the effect of this term upon $D_t v$, we get—

$$\Delta D_t v = \frac{2.778^m}{86400 \times 2 n \cos \theta}$$

With this expression, we get, on the parallel of 45° , with the value of n in § 14, $\Delta D_t v = 1.126^m$ per hour. Even this quantity would be of little consequence in estimating the approximate velocities of the general motions of the atmosphere; but the effect of the term in question must be very much less than this, since, in the slow interchanging motions between the equatorial and polar regions, the time in which a particle at rest in its extreme northern or southern position arrives at its maximum velocity must be very much more than 24 hours, as supposed above. The effect of this term then must be extremely small. Of course, this applies only to the slow general motions between the equatorial and polar regions, and not to those belonging to cyclonic disturbances of the atmosphere.

For the same reasons, the term $D_t^2 v$ is always very small, and in the general motions of the atmosphere may be neglected without sensible error. Equation (42) therefore furnishes us with a measure of the amount of friction when we know the value of $D u$.

If we put—

s = the motion of a particle of atmosphere;

F_s = the frictional resistance in the direction of that motion; and

i = the inclination of s to v , the eastward motion :

we have—

$$\begin{aligned} F_v &= F_s \cos i & F_u &= F_s \sin i \\ D_t v &= D_t s \cos i & D_t u &= D_t s \sin i \end{aligned}$$

By means of these equations and (42), putting $D_t^2 v = 0$, we get—

$$F_u = 2 n \cos \theta D_t u \tan i = 2 n \cos \theta D_t v \tan^2 i$$

With this value of F_u , we get from the preceding equations for the eastward component of velocity—

$$(43) \quad \dots \dots D_t v = \frac{\frac{1}{a'} D_u \log P' - g h \frac{D_u a'}{a'} + D_t^2 u}{2 n \cos \theta} \cos^2 i$$

Hence we get for the velocity of motion—

$$(43') \quad \dots \dots D_t s = \frac{\frac{1}{a'} D_u \log P' - g h \frac{D_u a'}{a'} + D_t^2 u}{2 n \cos \theta} \cos i$$

In the case of no friction, or where the effect of friction is neglected, we must put $\cos i = 1$.

When the direction of the wind, or, in other words, the value of i , is known from observations, the effect of friction in the preceding expressions is known; but when this direction is not known, we must neglect this effect, and put $\cos i = 1$. At the earth's surface, especially in the trade-wind zones, the value of i may be considerable; but here the value of $D_t v$ is small, and hence the effect of friction, and in the upper regions of the atmosphere where friction must be very much less, the value of i must be small, and $\cos i$ differ but little from unity.

43. In order to compute the expression of $D_t v$ (43), it is necessary to know first the value of a' , and this is obtained from (11) with the value of a_0 in (20), a' being the value of a at the surface of the earth, and hence the surface-values of t must be used in (11).

In the case of the mean annual temperatures, we can use the expression of t in (24), and with this we get from (11)—

$$(44) \quad \left\{ \begin{aligned} a' &= a_0 \left(0.968 + 0.007 \cos \theta + 0.079 \cos 2 \theta + 0.004 \cos 3 \theta + 0.007 \cos 4 \theta \right) \\ \frac{1}{a'} &= \frac{1}{a_0} \left(1.035 - 0.007 \cos \theta - 0.084 \cos 2 \theta - 0.004 \cos 3 \theta - 0.010 \cos 4 \theta \right) \\ \frac{D_\theta a'}{a'} &= \frac{-0.007 \sin \theta - 0.163 \sin 2 \theta - 0.012 \sin 3 \theta - 0.028 \sin 4 \theta}{1 + 0.082 \cos 2 \theta} \end{aligned} \right.$$

With the value of $\frac{1}{a_0} = g \times 7989^m$, obtained from (20), the second of these equations gives the values of $\frac{1}{a'}$ for any given value of the polar distance θ , and the last one gives the values of $\frac{D_\theta a'}{a'}$, from which we get the values of $\frac{D_u a'}{a'}$, or its equal $\frac{D_\theta a'}{r a'}$.

In the case of the January and July temperatures, the values of a' and $\frac{1}{a'}$ must be computed from (11) and (20) with the temperatures or values of t in (11), contained in Table V, for each tenth degree of latitude, interpolated to every fifth degree, and then, by means of the differences, the values of $D_\theta a'$ and of $\frac{D_\theta a'}{a'}$ can be approximately obtained.

In order to express the value of $D_u \log P'$ in terms of the gradients in Table X, we must put—

$$D_u \log P' = \frac{D_u P'}{P'} = \frac{1}{P'} \cdot \frac{G}{11111111}$$

In this way, the values of D_v have been computed from (43), putting $\cos^2 i = 1$, for each fifth degree of latitude so far as the data suffice for the purpose, except near the equator, where the formula is not applicable on account of the smallness of $\cos \theta$ in the denominator, and the results are given in the following table. The values of G and of $P' = B_0$ in Table X have been used for the purpose. The velocity D_v is given in kilometers per hour.

TABLE XI. .

Lat.	D_v .		
	Mean temperatures.	January.	July.
°	km.	km.	km.
+75	- 4.4 + 4.8 h.	- 1.9 + 4.4 h.	- 5.7 + 5.1 h.
70	- 3.3 6.6 "	- 1.1 7.5 "	4.8 5.8 "
65	+ 0.2 7.7 "	+ 1.6 9.6 "	- 1.2 5.9 "
60	3.9 8.3 "	5.5 10.9 "	+ 2.3 5.6 "
55	5.5 8.5 "	7.0 11.6 "	4.0 5.4 "
50	5.4 8.6 "	6.4 12.1 "	4.4 5.1 "
45	4.9 8.8 "	5.5 12.6 "	4.1 5.0 "
40	+ 2.6 9.0 "	+ 2.8 12.9 "	+ 2.4 4.8 "
35	- 1.2 9.3 "	- 1.0 13.8 "	- 1.4 4.6 "
30	8.6 9.5 "	9.1 14.7 "	8.1 4.3 "
25	14.4 9.4 "	16.0 15.2 "	12.2 3.6 "
20	15.1 9.0 "	20.2 15.6 "	11.7 2.4 "
+15	12.5 5.6 "	21.8 10.5 "	3.8 0.6 "
-15	25.0 8.2 "	18.7 4.9 "	31.0 11.7 "
20	20.9 7.8 "	18.8 5.8 "	23.0 10.0 "
25	-10.3 7.6 "	-10.4 6.6 "	-16.1 8.7 "
30	+ 3.8 7.5 "	+ 2.3 7.3 "	+ 5.2 7.8 "
35	12.4 7.4 "	10.0 7.8 "	14.4 7.1 "
40	18.7 7.4 "	16.0 8.2 "	21.4 6.7 "
45	24.0 7.5 "	21.0 8.6 "	27.0 6.4 "
50	27.5 7.5 "	24.5 8.9 "	30.5 6.1 "
55	27.3 + 7.5 "	+24.6 + 9.1 "	+30.0 + 5.9 "
-60	+21.9		

44. Since h vanishes at the earth's surface, the first term in the expression of D_v in this table represents the surface-velocity of the east or west component of motion at the surface. This is a little less than what is usually obtained from the gradients; for the barometric pressures having been reduced to the gravity of the parallel of 45° , the gradients in the middle latitudes are diminished on this account about 0.07^m , which, by (21'), for the temperature of 0° , corresponds on the parallel of 45° to 2.16^m per hour. This shows the importance of having all barometric pressures reduced to a fixed measure instead of being expressed in measures which vary with the latitude.

Since h vanishes at the earth's surface, the easterly component of the velocity of the wind there is represented by the first term in the expression of D_v in Table XI. Between the parallels of 36° and 66° in the northern hemisphere, and south of 29° in the southern hemisphere, as far as barometric observations have been made, the winds, according to the results of this table, have an easterly component, and between the parallels of 36° N. and 29° S. there is a westerly component. North of the parallel of 66° N. there is also a westerly component. By a glance at Plate 3 of the late Prof. Coffin's great work on the "Winds of the Globe," it is seen that this is exactly in accordance with observation, except that the dividing line between the two systems of winds in the southern hemisphere is a little south of the parallel of 29° . But the observations in this zone are not sufficient for determining the position of this line very accurately, and the position obtained from the barometric gradients is, without doubt, the more correct one.

Again, according to the authority of Mr. Laughton,* "In both hemispheres to the north or south of the parallels of 35° or 40° , a strong westerly wind blows with great constancy all around the world.

*Physical Geography in its Relation to the Prevailing Winds and Currents, p. 101.

In the southern hemisphere, more particularly, it blows with a persistence little less than that of the trade-winds, but with a strength which, although fitful, is very much greater. From a fresh, strong breeze, it rises frequently into a violent gale, and, as such, blows for days together, the mean direction being nearly west, from which it seldom varies more than a couple of points on either side. South of the Atlantic, south of the Indian Ocean, south of Australia, in the higher latitudes of the Southern Pacific, and to the southward of Cape Horn, we find it still the same, a westerly gale, whose strength and constancy combined have enabled Australian clippers to make passages which seem to border on the fabulous. In the northern hemisphere, it has not the clear range which it has in the southern; but there, too, it prevails in the most decided manner."

From the results of Table XI, it is not only seen that the velocities in the higher latitudes of both hemispheres are eastward, but that they are very much greater in the southern than in the northern hemisphere, which is exactly in accordance with observation according to the preceding quoted paragraph.

The smallness of the eastward components of velocity in the northern hemisphere compared with those of the southern is due to the greater amount of land and mountain-ranges in the northern hemisphere than in the southern, which increases the resistances to the eastward motions, and consequently the greater eastward motions of the southern hemisphere, by means of the deflecting force arising from the earth's rotation, causes a greater depression there, and a great part of the atmosphere to be thrown into the northern hemisphere. This also accounts for the mean position of the equatorial calm-belt being, in general, a little north of the equator. For the same reason, the tropical calms of the northern hemisphere are farther from the equator than those of the southern hemisphere.

The easterly components for the northern hemisphere are known, from observation, to be very small; but estimates of the actual velocities have been made but for a very few places around the globe. Prof. Coffin obtained* for the average velocity for the whole of the United States, estimated by the Smithsonian scale, 2.0 miles (3.2^{km}) per hour, with a resultant direction nearly from the west, making the eastward component sensibly the same. This result agrees almost precisely with the result given for the mean of the year in Table XI for the average latitude of the United States; but it must be remembered in these comparisons that the theoretical results are those of the average all around the globe, and that various disturbing causes, to be considered hereafter, may cause considerable deviations from this average in different longitudes on the same latitude. The effect of friction at the surface, which has been neglected in the formula (43), is to make the actual velocities less than those given in Table XI by the formula.

The westward velocities in the trade-wind zones, as given in Table XI, are greater in the southern than the northern hemisphere. This is in accordance with observations of the trade-winds of the Atlantic Ocean, on which the southeast trade-winds are observed to be much stronger than the northeast ones. The theoretical results, however, in these zones are probably considerably too large on account of the neglect of the friction term in (43), which in these zones is very much magnified near the equator from the smallness of $\cos \theta$ in the denominator of the formula.

The parallels of 36° in the northern and of 29° in the southern hemisphere, being parallels on which there is no east or west motion on the average around the globe, and where, on account of there being no barometric gradient, there can be no north or south component of velocity, represent the mean position of belts in which calms abound, called tropical calm-belts.

45. From a comparison of the eastward and westward velocities at the earth's surface in both hemispheres, contained in Table XI, it is seen that the winter velocities in each hemisphere are greater than those of summer. We have but few results deduced from observations to confirm this result, but it is completely confirmed so far as they go. From Prof. Coffin's "Winds of the Globe" (pp. 648-653), we extract the following mean directions and true velocities in mean direction for winter and summer, and have computed the corresponding components of velocity contained in the following table.

* Winds of the Globe, p. 641.

TABLE XII.

Region or place.	Mean direction.		True velocity.		East component.		South component.	
	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.
	°	°	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.
Red River Settlement.....	S. 72 W.	S. 85 W.	0.60	1.12	+ 0.57	+1.12	- 0.18	-0.10
Pacific Coast.....	S. 3 W.	N. 80 E.	1.64	1.51	0.09	1.48	- 1.64	+0.26
Northern Lake Region.....	N. 59 W.	S. 55 W.	2.32	1.73	2.05	1.42	+ 1.23	-0.99
Canada and Nova Scotia.....	S. 87 W.	S. 64 W.	3.49	2.30	3.49	2.07	- 0.18	-1.01
New England States.....	N. 52 W.	S. 66 W.	5.41	2.67	4.26	2.44	+ 3.33	-1.09
Region of the Missouri.....	N. 73 W.	S. 37 W.	2.26	1.69	2.16	1.02	+ 0.66	-1.35
South of the Great Lakes.....	S. 77 W.	S. 68 W.	2.86	2.13	2.79	1.98	- 0.64	-0.80
Indiana, Illinois, and Ohio.....	S. 72 W.	S. 69 W.	2.64	1.67	2.51	1.56	- 0.82	-0.60
New York to North Carolina west of Appalachian Range.....	S. 79 W.	S. 77 W.	3.52	2.04	3.45	1.99	- 0.67	-0.46
Middle States east of the Appalachian Range.....	N. 72 W.	S. 69 W.	3.42	1.67	3.25	1.56	- 1.05	-0.60
Kentucky and Tennessee.....	S. 66 W.	S. 79 W.	2.35	1.00	2.15	0.98	- 0.96	-0.19
Atlantic coast (latitude 31° to 38°).....	N. 70 W.	S. 25 W.	2.68	1.82	2.52	+0.77	+ 0.92	-1.65
Texas (latitude 30°).....	N. 3 W.	S. 43 E.	3.59	3.78	0.19	-2.57	- 3.59	-2.76
Gulf States.....	N. 45 W.	S. 22 E.	1.25	1.00	0.88	-0.38	+ 0.88	-0.94
Northern Florida.....	N. 15 W.	S. 5 W.	1.78	1.47	+ 0.46	+0.13	+ 1.72	-1.46
Salt Ponds of Florida (latitude 25°).....	N. 30 E.	S. 70 E.	8.11	8.84	- 0.40	-8.60	+ 0.70	-2.13
City of Mexico.....	S. 9 W.	N. 78 E.	3.19	0.88	+ 0.50	-0.86	- 3.16	+0.18
Catharina Sophia, Guiana.....	N. 58 E.	N. 83 E.	8.73	4.86	- 7.41	-4.83	+ 4.63	+0.59
Horta Fayal, Azores.....	S. 78 E.	S. 43 W.	1.63	2.14	- 1.59	+1.45	- 0.34	-1.56
Sandwich Manse, Orkney Islands.....	S. 62 W.	S. 62 W.	6.33	2.86	+ 5.59	+2.52	- 3.00	-1.34
Port Foulke, Arctic Ocean.....	N. 42 E.	S. 82 W.	17.21	0.47	-11.51	+0.46	+12.78	-0.06
Port Kennedy, Arctic Ocean.....	N. 44 W.	N. 31 W.	13.08	8.74	+ 9.10	+7.60	+ 9.40	-4.40

By comparing the east components of winter and summer above, it is seen that throughout the United States, except on the Pacific coast and in the extreme southern States, and at the Orkney Islands, the winter component is in every case greater than the summer component. The extreme southern States are so near the parallel of the calm-belt that the velocities in the direction of the resultant, and the direction itself, are so uncertain that the ratios between the components for winter and summer are very various, and the signs of the components even change in some cases.

46. The effect of the second term in the expression of D_v in Table XI is to increase very much the velocities of the eastward motions in the upper strata of the atmosphere over those at the surface in the higher latitudes; and, in the trade-wind zone, the westward velocities near the surface are at a very moderate elevation changed to eastward velocities. Thus, on the parallel of 45° , and at an elevation of 5 kilometers, we have, for the mean temperature of the year, $D_v = 4.9^{\text{km}} \times 8.8^{\text{h}} = 48.9^{\text{km}}$ per hour, and at greater elevations this velocity becomes still much greater. On the parallel of 25° , we have D_v at the surface equal to 14.4^{km} west, and at the elevation of only 3^{km} it becomes 13.8^{km} east, and hence at the elevation of only about 1.5^{km} there is no motion east or west. These results are in accordance with observation, for travelers have experienced a strong westerly wind, at great elevations, on Mauna Loa, on the passes of the Rocky Mountains and the Andes, on the top of Pike's Peak and Mount Washington, on the Peak of Teneriffe, and at every very elevated position in either hemisphere all around the globe, except in the calm-belt near the equator, even when there is an easterly wind in the same latitude in less elevated positions. It is seen from Table XI that in the trade-wind zones, where at the surface the wind is easterly, at a very moderate elevation there is a very strong current from the west; and this is especially the case in the winter. This accounts for the transportation of volcanic ashes through long distances eastward when in the trade-wind belt at the surface there was a strong current from the east. On the 1st of May, 1812, the island of Barbadoes was suddenly obscured by a dense cloud, and its surface quickly covered by a shower of ashes from an eruptive volcano of St. Vincent, more than a hundred miles to the westward. Also, on the 20th of January, 1835, the volcano of Cosequina, lying in the belt of the northeast trade-winds, sent forth great quantities of lava and ashes, and the latter were borne in a direction just contrary to the surface wind, and lodged on the island of

Jamaica, 800 miles to the northeast. This latter happened at a season when, according to the results of Table XI, the upper currents have a very great easterly tendency, and hence at a time very favorable for the transportation of the ashes to so great a distance. The great eastward velocities of the upper currents is also established by the observations of the clouds, especially of the cirrus clouds, which are supposed to have an altitude generally of 7 or 8 kilometers. The eastward velocity of these clouds has been estimated at times to be as much as 120 miles (193^{km}) per hour, and it rarely happens that they have no eastward tendency. The average, therefore, according to this somewhat uncertain kind of observations, may be put at nearly 100^{km}. This eastward velocity is a little greater than that given by Table XI at the height of the cirrus clouds in the middle latitudes for the mean temperature of the year, but corresponds with that belonging to January.

The eastward motion of the atmosphere in the latitude of the trade-winds is also confirmed by observations made on the directions of the clouds at Colonia Tovar, Venezuela, latitude 10° 26', as given in the Report of the Smithsonian Institution for 1857 (p. 254). While the motion of the lower clouds was in general from some point toward the east, the observed motion of nearly all the higher clouds was from some point toward the west.

51. From a comparison of the expressions of $D_t v$ in Table XI, for January and July, it is seen that the eastward velocities of the upper strata of the atmosphere are very much greater in winter than in summer. On the parallel of 45°, in the northern hemisphere, we have for an altitude of 5^{km} $D_t v$ equal to 66.1^{km} in January and equal to only 29.1^{km} in July. Results deduced from the discussion of observations made at elevated places for both seasons are needed for comparison with this theoretical result.

On the parallel of 25°, in the northern hemisphere, the height at which there is no east or west motion of the atmosphere by Table XI is 1.1^{km} in January, but 3.4^{km} in July. North of this parallel, these heights, where there is no east or west motion, gradually diminish until at the parallel of 36°, the parallel of the tropical calm-belt, this plane touches the surface. Hence there is an oscillation with the seasons of the height of this plane dividing the west or southwest from the east or southeast winds. This explains the winds of the Peak of Teneriffe, which at the top blow in general from the southwest, while lower down they blow alternately from the northeast and southwest, changing with the seasons. In the summer season, the dividing plane between the two systems of winds blowing in nearly contrary directions, is highest, and is found on the Peak of Teneriffe from observation to be about 1.5 miles high. Prof. Piazzzi Smyth, who spent several months on the peak making astronomical observations, on leaving his station at Alta Vista, at the height of 10,700 feet above the level of the sea, on the 25th of August, experienced a southwest breeze, but at an altitude of 6,700 feet it changed to one from the northeast. Returning again on the 30th of August, he experienced a similar change at the same height, the strength of the wind increasing as he ascended, and blowing from the southwest at Alta Vista, as when he left.

47. Since each upper stratum acts upon the one beneath it by means of friction, and that upon the next one, and so on, down to the earth's surface, the force which overcomes the friction between the atmosphere and the earth's surface is the sum of all the deflecting forces depending upon the earth's rotation and upon inertia, and putting F_v' for F_v at the earth's surface, we get from (33)—

$$F_v' = \int_m (2n \cos \theta D_t u - D_t^2 v)$$

in which m is the mass of the atmosphere. But since the condition of continuity must be satisfied, we must have, at any latitude, just as much air moving south as north, and hence for the whole atmosphere from the surface to the exterior we must have—

$$\int_m D_t u = 0$$

The first term, therefore, of the preceding equation vanishes, and we have—

$$(45) \dots \dots \dots F_v' = - \int_m D_t^2 v$$

For any unit of surface, this must be integrated with reference to the mass of a column of atmosphere having this unit for a base. Now, we have seen that the eastward velocities of the

atmosphere are much greater above than below, and hence, as the atmosphere of the upper currents approaches the poles, it gradually sinks down, to return, toward the equator, nearer the surface, and D_v , in order to satisfy the conditions, must gradually decrease, and in this case the last member of (45) is positive, that is, the sum of the forces tends to overcome the frictional resistance at the earth's surface to the eastward motion of the atmosphere, and the observed amount of motion depends upon these forces. In the equatorial regions, where the atmosphere ascends, of course the reverse of this is the case, and we have these forces tending to overcome the frictional resistance at the earth's surface to the westward motions there. The force, then, which overcomes the frictional resistances, at the earth's surface, to the eastward motion of the atmosphere, depends upon the sum of the moments of inertia, or of the amounts of velocity lost, of the particles of atmosphere, in passing toward the poles, in the upper regions, where the eastward velocity is greater, and returning toward the equator nearer the earth's surface, where the eastward velocity is less. The force, likewise, which overcomes the resistances to the westward motions at the surface in the zones of the trade-winds, is the moment of inertia lost in the decreasing westward velocities of the atmosphere as it ascends and commences to return in the upper regions toward the poles.

48. The eastward velocity D_v in Table XI is that which satisfies the conditions of the problem in the case of no friction, in which case D_u , the interchanging velocity between the equatorial and polar regions, vanishes, and consequently the deflecting force overcoming friction in the case of friction. Where there is friction there must be a force to overcome it, and hence D_u must have a value, that is, there must be a gradual interchange of atmosphere between the equatorial and polar regions, and this motion must be just sufficient to give rise to a force sufficient to overcome the friction and the inertia of the eastward motions. If the velocity of the motion of the upper strata of the atmosphere toward the poles were a little too great, it would give rise to an increased eastward velocity, which, by means of the deflecting force depending upon the earth's rotation (§ 13), would at once diminish the velocity, and reduce it to that which satisfies the conditions of the problem.

The difference between the atmospheric pressure at the equator and the poles, at the earth's surface, depends almost entirely upon the eastward motion, or value of D_v , at the earth's surface, as may be seen from the first of (15), since h in the last term vanishes at the surface, and consequently this difference is entirely independent of the difference of temperature, which affects only the last term of this equation. Hence, in the northern hemisphere, where there is a difference of 30° or more in the higher latitudes, between winter and summer, the difference of barometric pressure amounts to only about 2^{mm}. The reason of this is that the increased eastward velocities in winter, as shown in Table XI, give rise to a force depending upon the earth's rotation by the principle of § 13, which tends to prevent the flow of the atmosphere in the upper regions from the equatorial to the polar regions, as the volume there is contracted by diminution of temperature, and the height of the atmosphere is decreased toward the poles, nearly in the same proportion that its density is increased by the diminution of temperature, so that there is scarcely any change in the mass of the atmosphere, and consequently in the barometric pressure, between winter and summer.

If the earth had no rotation on its axis, this deflecting force depending upon its rotation would not exist, and then, if this great difference of temperature between the equator and the poles could be maintained, there would be a very great difference of pressure and a very rapid interchanging motion between the equatorial and polar regions.

